COMPUTATIONAL SOLID MECHANICS

Assignment 1

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Contents

Part I - Rate Independent Models

1.1 Introduction

Material properties used to study rate independent models are summarized in Table 1.1.

Table 1.1: Material properties

Property	Values
Young modulus (E)	20000
Poisson ratio (ν)	0.3
Hardening/softening modulus (|H|)	0.2
Yield stress (σ_u)	200
Ratio of compression and tension strength (n)	3

To assess the correctness of the implementation, 3 loading paths summarized in Table 1.2 were applied.

Table 1.2: Input data for loading paths

Cases	$\Delta\sigma_1^{(1)}$	$\Delta\sigma_2^{(1)}$	$\Delta\sigma_1^{(2)}$	$\Delta\sigma_2^{(2)}$	$\Delta\sigma_1^{(3)}$	$\Delta\sigma_2^{(3)}$
Loading path 1	300	0	-1100	0	400	0
Loading path 2	300	0	-800	-800	-400	-400
Loading path 3	300	300	-1300	-1300	800	800

1.2 Results

Stresses and parameters such as damage variable obtained from the calculations are described in the following subsections.

1.2.1 Linear vs exponential hardening/softening law

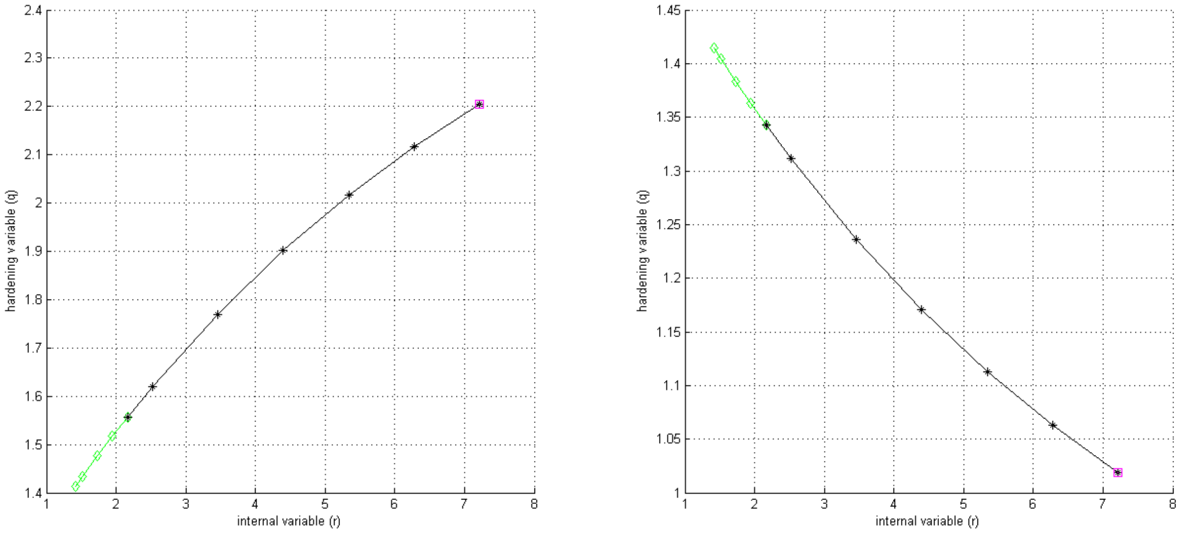
Equation 1.1 was already implemented in the program while Equation 1.2 was added in this exercise to consider the exponential relation between hardening variable (q) and internal variable (r).

$$q(r) = r_0 + H(r - r_0) \tag{1.1}$$

$$q(r) = q_\infty - (q_\infty - r_0)e^{A(1-\frac{r}{r_0})} \tag{1.2}$$

In this exercise, A was set to equal to absolute value of H and q_∞ was set to equal to twice of r_0 for the hardening model and equal to half of r_0 for the softening model .

The plots in Figure 1.1 are the results when considering symmetric damage model and applying loading path 3. The exponential relation between hardening variable (q) and internal variable (r) can clearly be seen for both hardening and softening models.



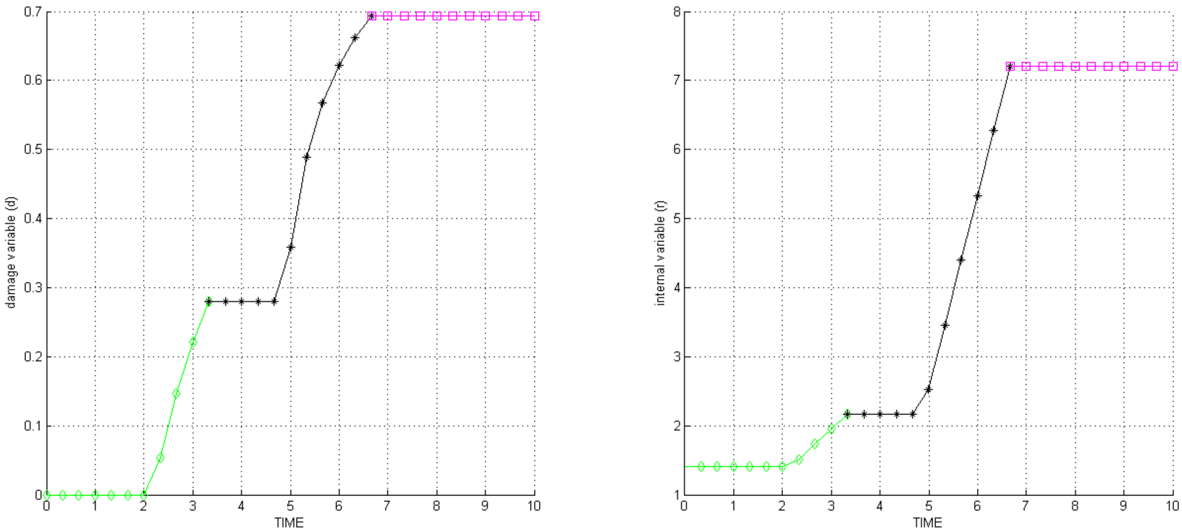
(a) Hardening model (b) Sofenting model

Figure 1.1: Result of implementing exponential hardening/sofening law

1.2.2 Damage variable

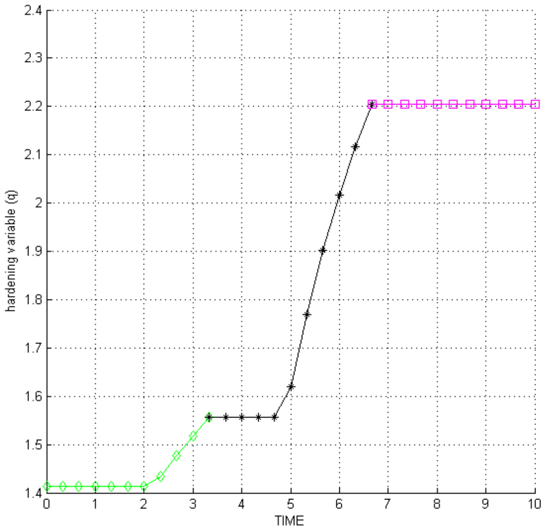
Damage variable (d) shown in Figure 1.2a was computed from hardening variable (q) and internal variable (r). It is as expected that the damage variable can never decrease same as the internal variable shown in Figure 1.2b. For the hardening variable, it can be either be increased or decreased depending on how the material would behave, hardened or softened, after reaching its elastic capacity as shown in Figure 1.3. Results in Figure 1.2 and Figure 1.3 are from the cases of symmetric damage model corresponding to loading path 3 and exponential hardening/softening behavior.

Note that, in all plots, the green, black and magenta portion of the plot represent the first, second and third segment of the loading path, respectively.

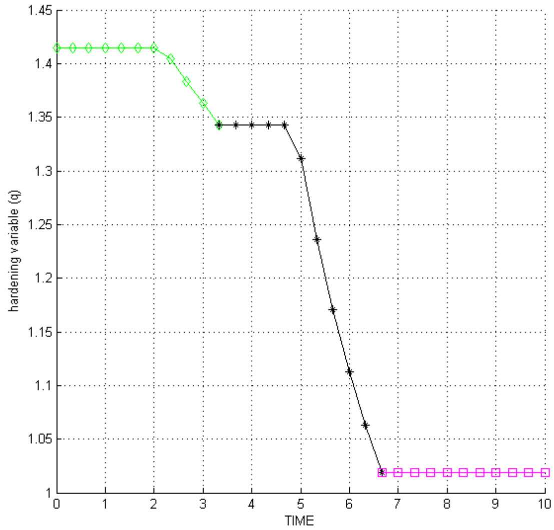


(a) Damage variable (b) Internal variable

Figure 1.2: Example of the obtained damage variable and internal variable



(a) Hardening model



(b) Softening model

Figure 1.3: Example of obtained hardening variable

1.2.3 Stresses

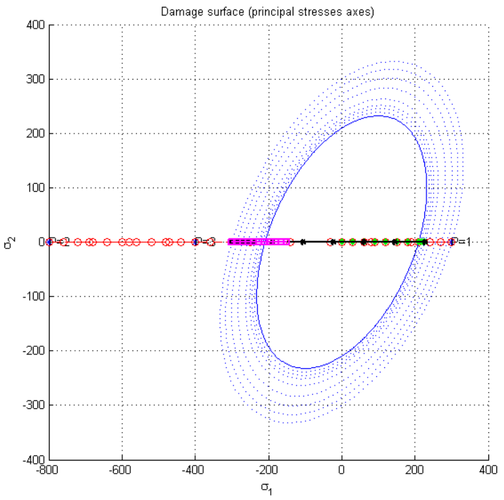
In order to study the non-symmetric tension-compression damage model and the tension-only damage model, the following equations were used as the basis for the algorithms. The modified routines can be found in the appendix.

$$\tau_\sigma = [\theta + \frac{1-\theta}{n}]\sqrt{\sigma : C^{-1} : \sigma} \tag{1.3}$$

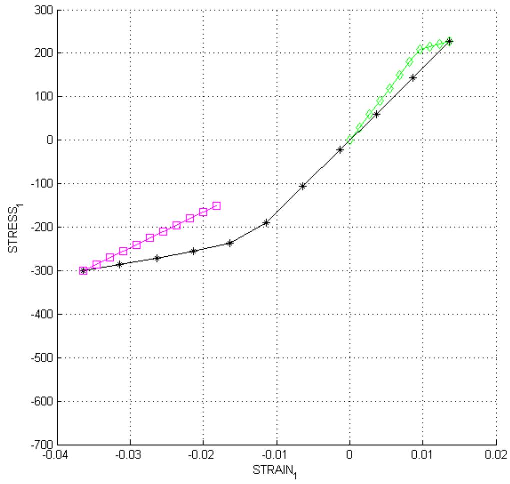
$$\tau_\sigma^+ = \sqrt{\sigma^+ : C^{-1} : \sigma} \tag{1.4}$$

The loading magnitudes summarized in Table 1.2 were chosen such that the difference of material behavior in the compressive state can be distinguished in each model. The plots in Figure 1.4 are the case corresponding to loading path 1 and exponential hardening. It can be seen that the main difference between three damage models is in the second segment of the loading path. By applying the same compressive loading in the second segment, the material reaches its elastic capacity first in the symmetric damage model, followed by the non-symmetric damage model, while the material stays elastic in the tension-only damage model. The same behavior can be seen in Figure 1.5 to Figure 1.9 for the cases corresponding to loading path 2 and loading path 3 with exponential hardening and the cases with exponential softening.

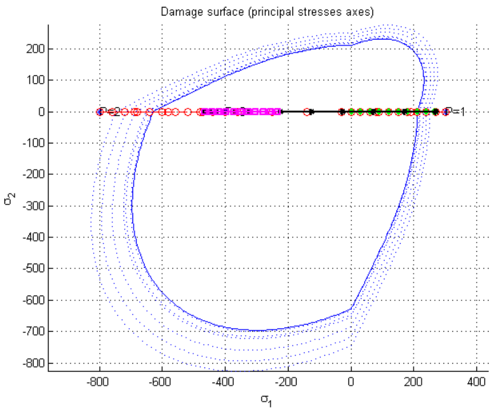
Note that for the plots in Figure 1.4 to Figure 1.6, the red line with circle markers represents the loading path while the green, black and magenta portion of the plot represent the calculated stress path corresponding to the first, second and third segment of the loading path, respectively.



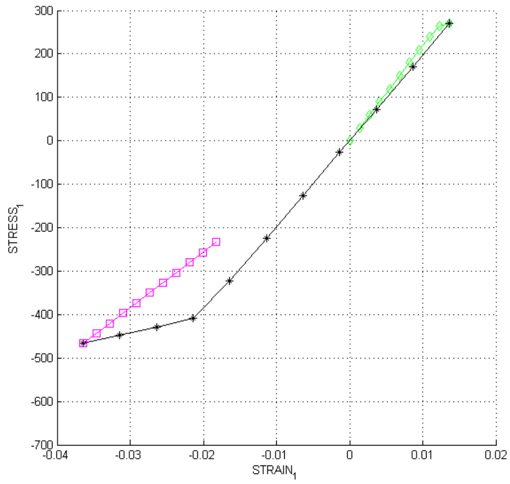
(a) Stress path - symmetric model



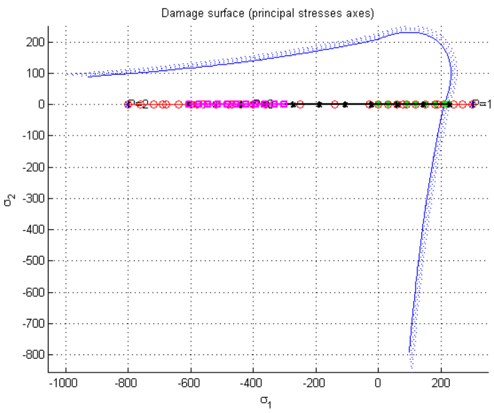
(b) Stress-strain curve - symmetric model



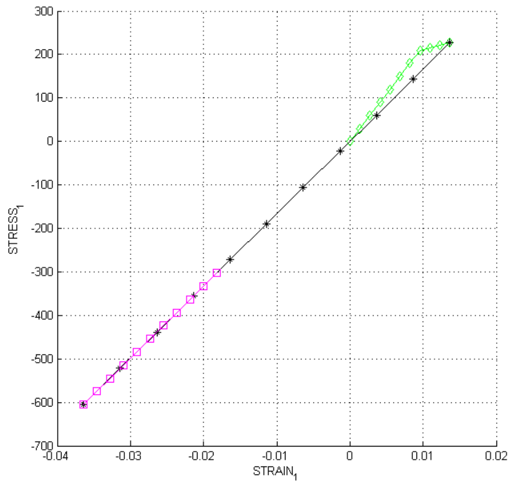
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model

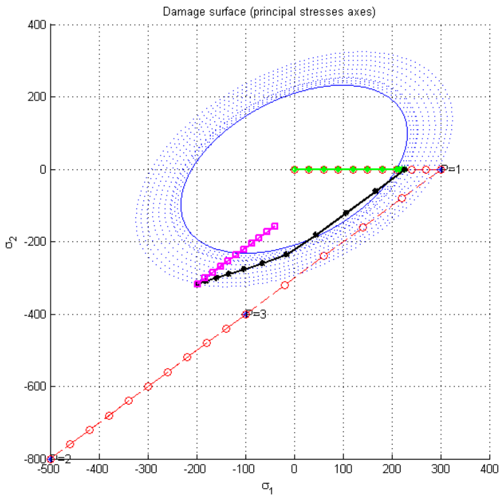


(e) Stress path - tension-only model

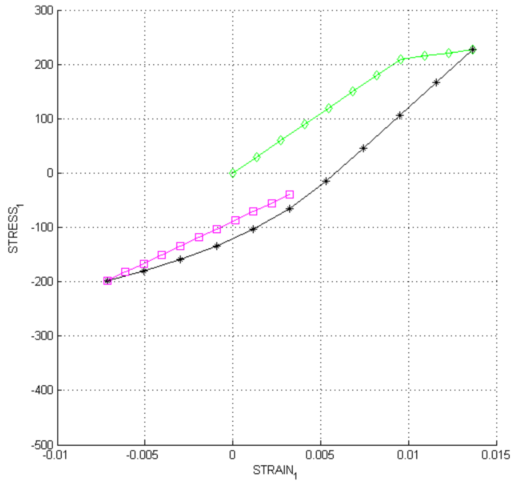


(f) Stress-strain curve - tension-only model

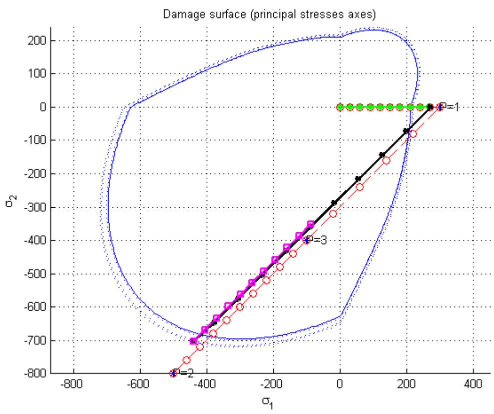
Figure 1.4: Stresses corresponding to loading path 1 and exponential hardening



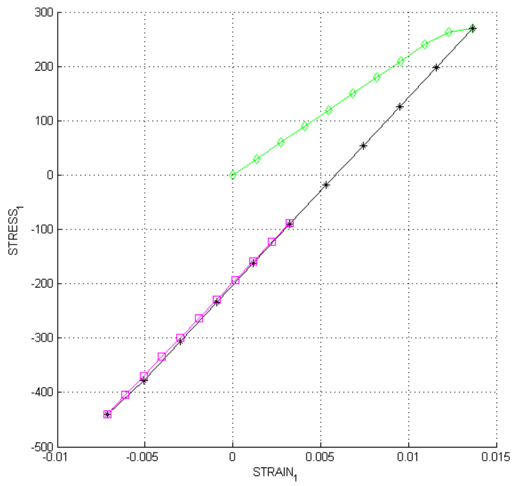
(a) Stress path - symmetric model



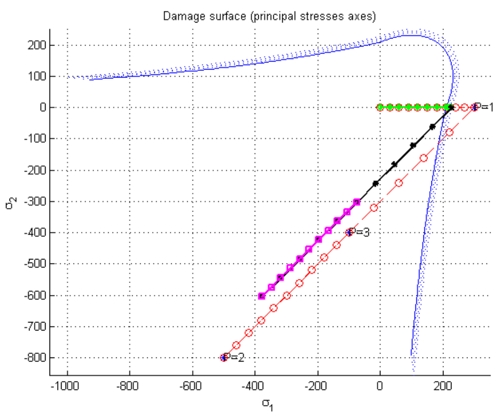
(b) Stress-strain curve - symmetric model



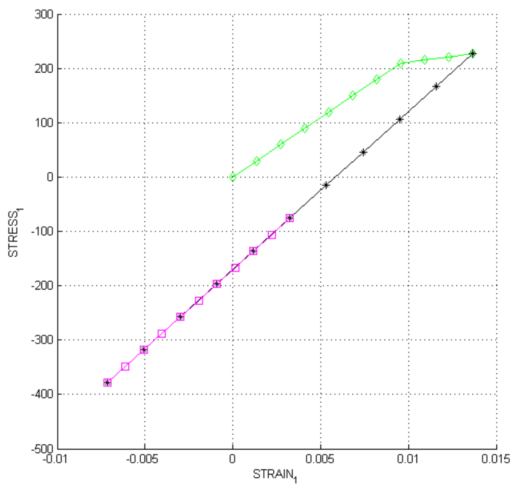
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model

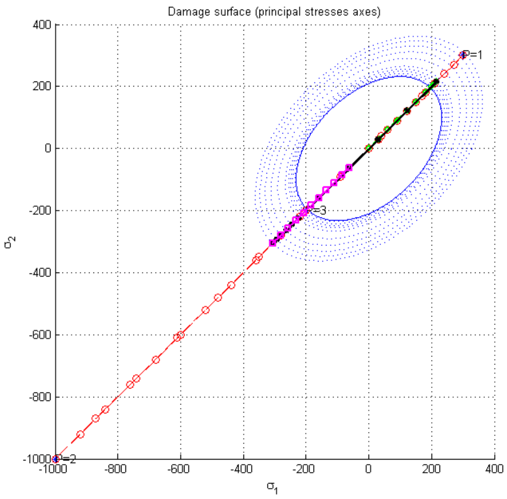


(e) Stress path - tension-only model

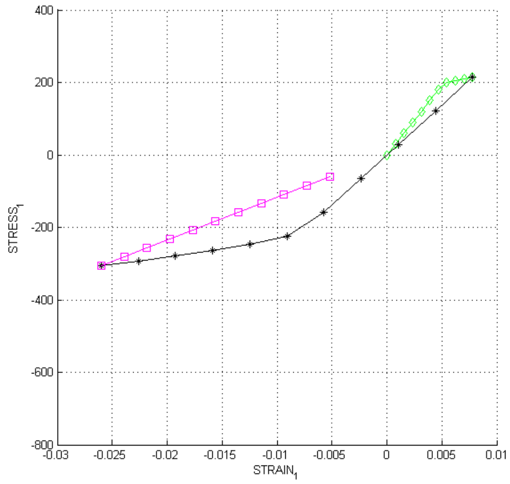


(f) Stress-strain curve - tension-only model

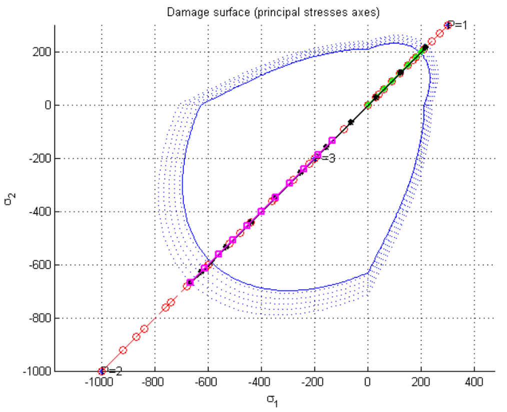
Figure 1.5: Stresses corresponding to loading path 2 and exponential hardening



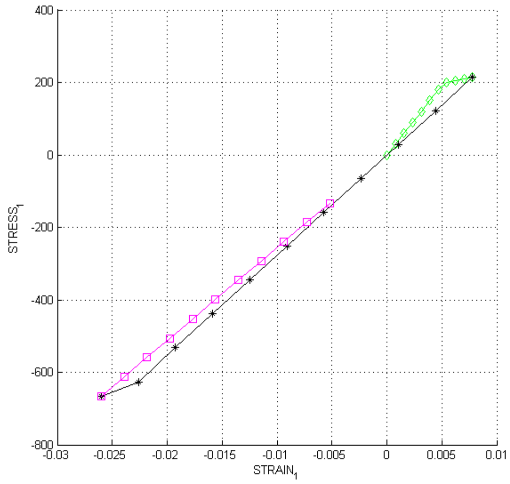
(a) Stress path - symmetric model



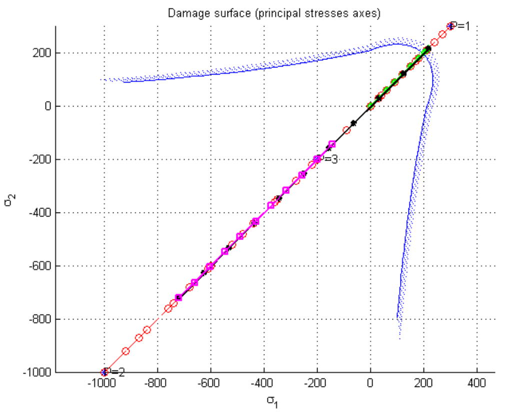
(b) Stress-strain curve - symmetric model



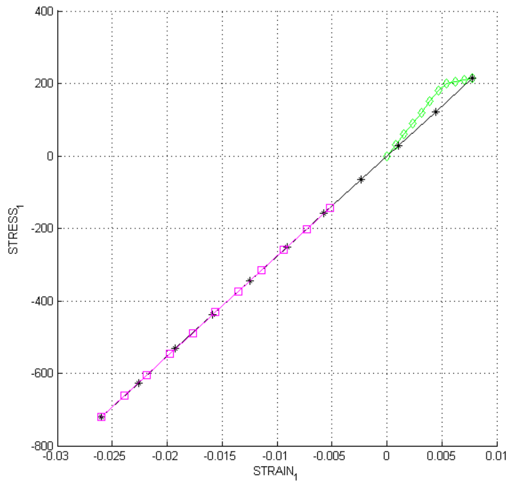
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model

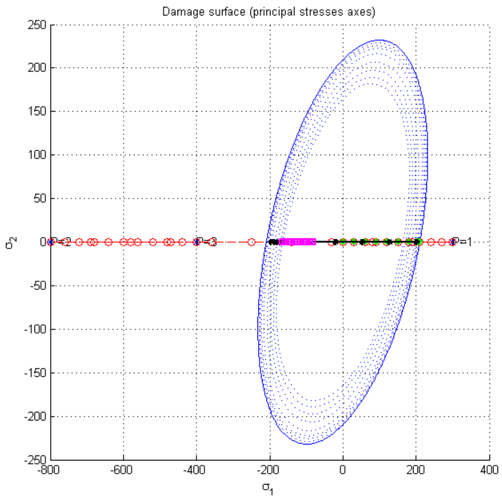


(e) Stress path - tension-only model

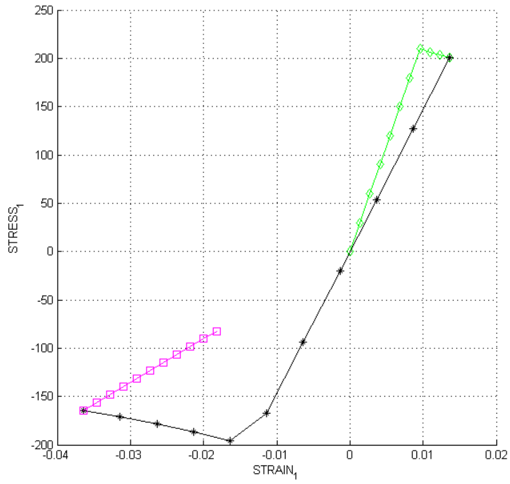


(f) Stress-strain curve - tension-only model

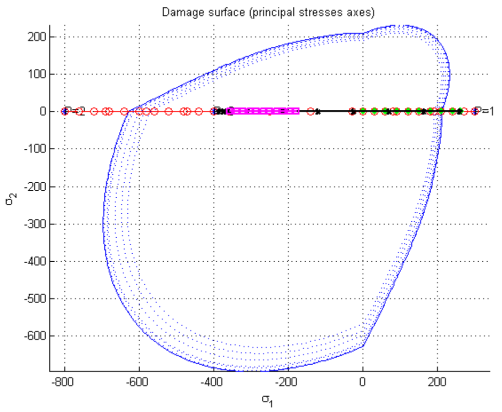
Figure 1.6: Stresses corresponding to loading path 3 and exponential hardening



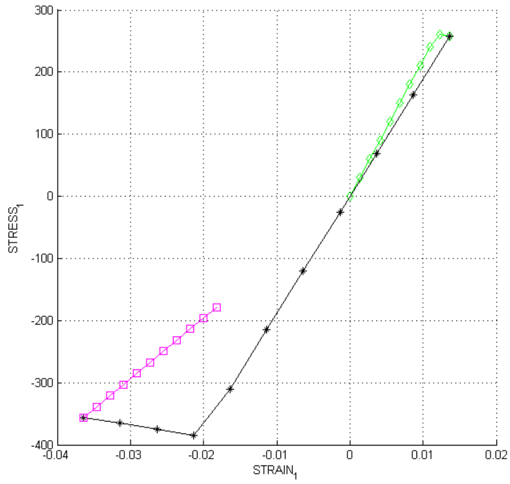
(a) Stress path - symmetric model



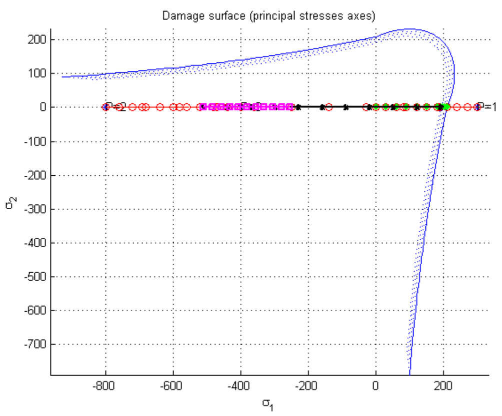
(b) Stress-strain curve - symmetric model



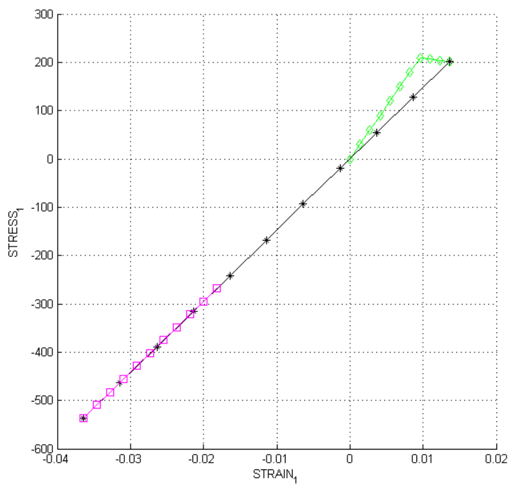
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model

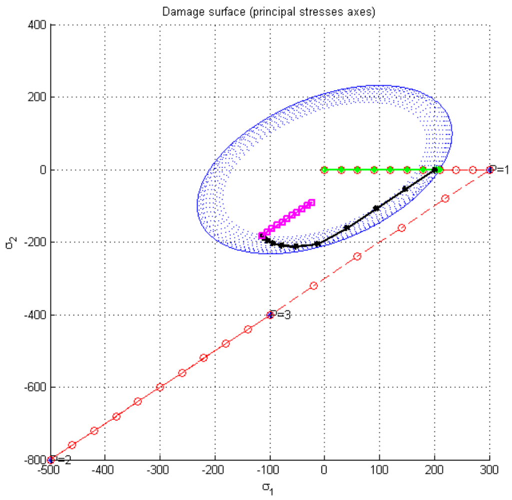


(e) Stress path - tension-only model

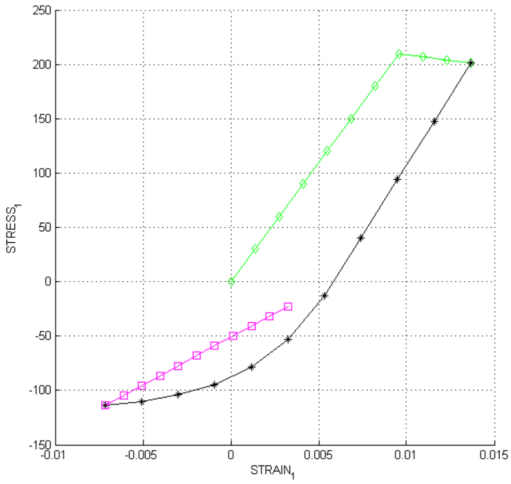


(f) Stress-strain curve - tension-only model

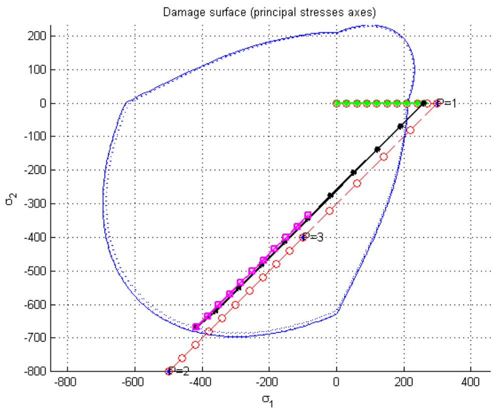
Figure 1.7: Stresses corresponding to loading path 1 and exponential softening



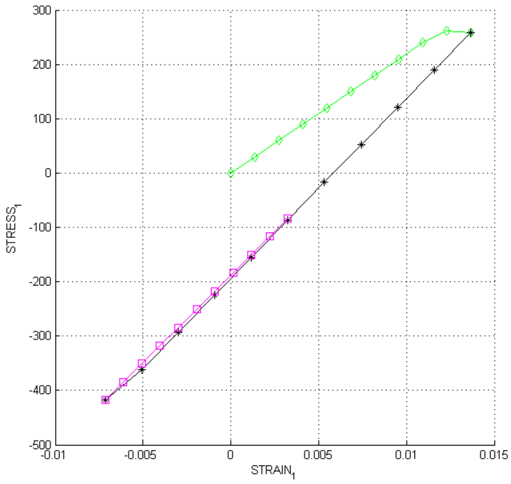
(a) Stress path - symmetric model



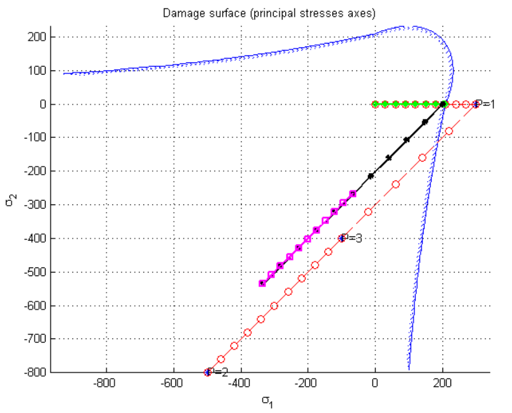
(b) Stress-strain curve - symmetric model



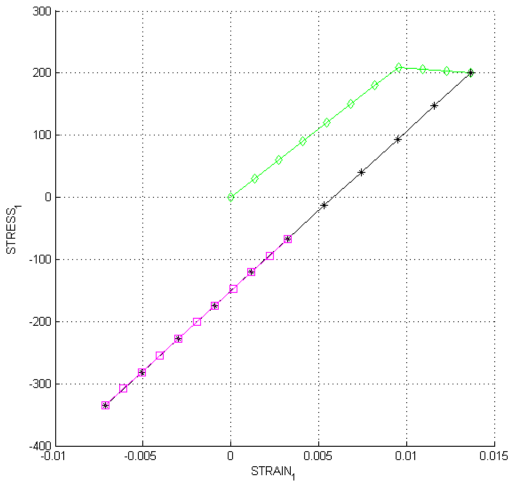
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model

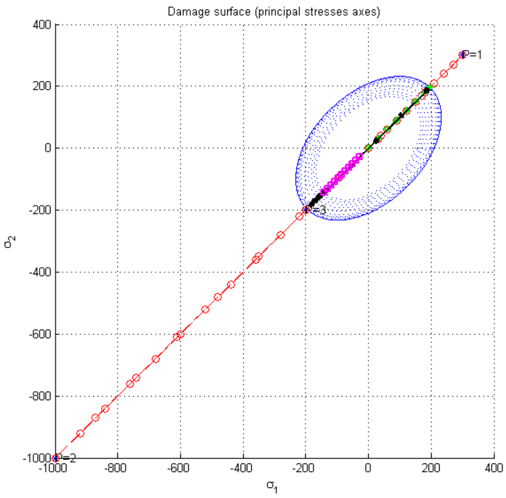


(e) Stress path - tension-only model

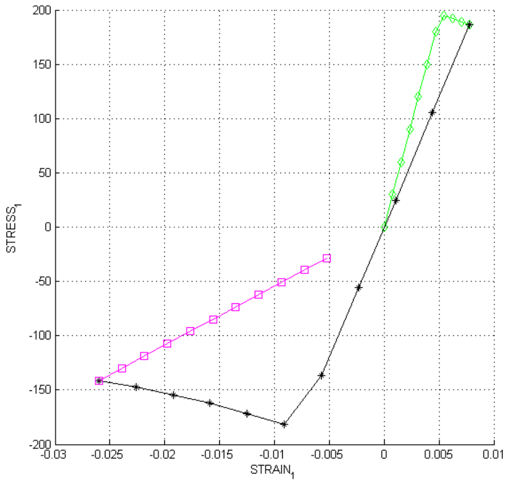


(f) Stress-strain - tension-only model

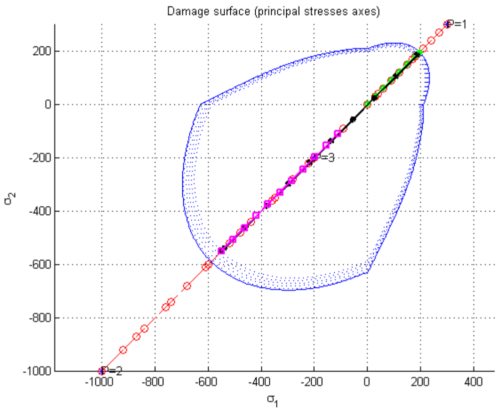
Figure 1.8: Stresses corresponding to loading path 2 and exponential softening



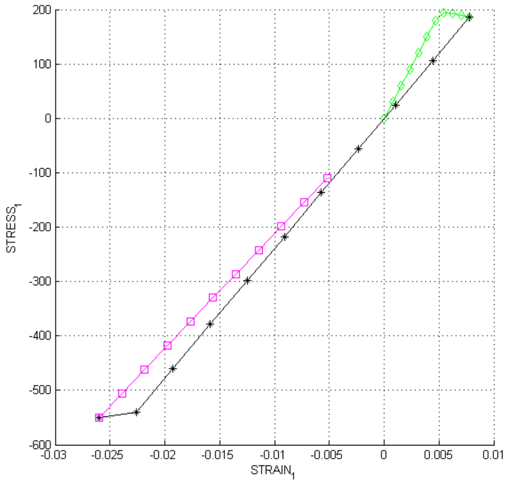
(a) Stress path - symmetric model



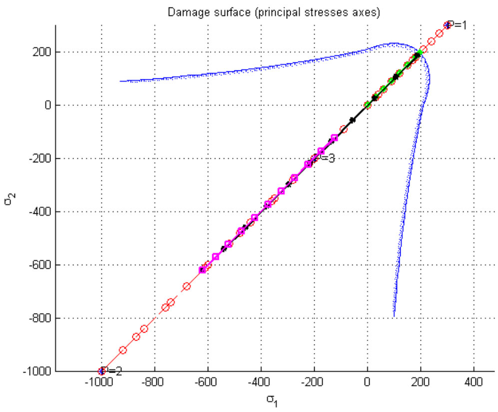
(b) Stress-strain curve - symmetric model



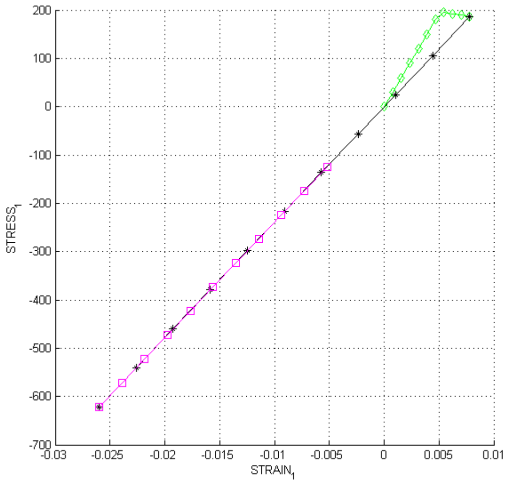
(c) Stress path - Non-symmetric model



(d) Stress-strain curve - non-symmetric model



(e) Stress path - tension-only model



(f) Stress-strain curve - tension-only model

Figure 1.9: Stresses corresponding to loading path 3 and exponential softening

Part II - Rate Dependent Models

2.1 Introduction

The simple uniaxial loading, loading path 1 in Table 1.2, was used to study rate dependent models. Symmetric damage model and linear hardening law were considered in the calculations. Material and numerical properties used in the calculations are summarized in Table 2.1. The values of $\eta = 0.3$, $\alpha = 0.5$ and total time = 10 were considered as the base case, unless stated otherwise.

Table 2.1: Material and numerical properties

Property	Values
Young modulus (E)	20000
Poisson ratio (ν)	0.3
Hardening/softening modulus (|H|)	0.2
Yield stress (σ_u)	200
Ratio of compression and tension strength (n)	3
Viscous coefficient (η)	0.1, 0.3, 0.9
Integration coefficient (α)	0, 0.25, 0.5, 0.75, 1
Total time	2, 3, 10

2.2 Results

Stresses and parameters such as damage variable when considering viscous effect are described in the following subsections. Sensivity studies regarding to viscous coefficient and integration coefficent are also included.

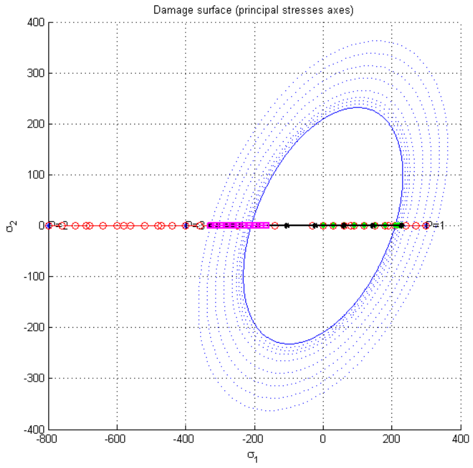
2.2.1 Stress paths

One obvious difference between rate independent and rate dependent models can be found in the stress path. Figure 2.1a shows the stress path of the case considering symmetric damage model without viscous effect while Figure 2.1b shows the stress path of the case considering symmetric damage model with viscous effect using $\eta = 0.9$. It can be seen in Figure 2.1b that the magenta portion of the plot is now extended outside the ellipsoids. That means the stresses are now allowed to be outside the damage surface when including the viscous effect.

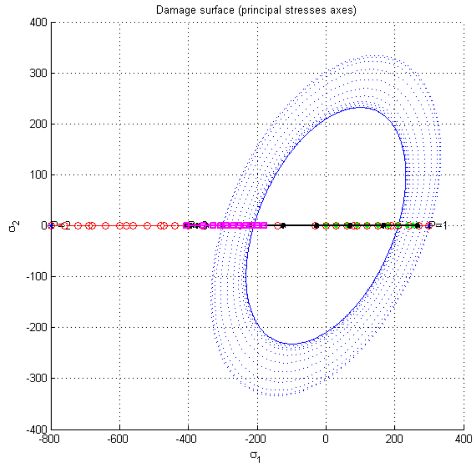
Note that the red line with circle markers represents the loading path while the green, black and magenta portion of the plot represent the calculated stress path corresponding to the first, second and third segment of the loading path, respectively.

2.2.2 Variation of viscous coefficient and strain rate

Viscosity and strain rate have similar effects on the stress-strain curves. If viscosity or strain rate increase, the stresses also increase. It can be seen that the maximum stress in Figure 2.2d which considers $\eta = 0.9$ is obviously larger than those in Figure 2.2a, Figure 2.2b and Figure 2.2c which either consider no viscous effect or smaller viscous coefficients. The effect of strain rate shows in Figure 2.3. Shorter total time provides higher strain rate which in turn gives larger maximum stress as seen when comparing Figure 2.3a and Figure 2.3b

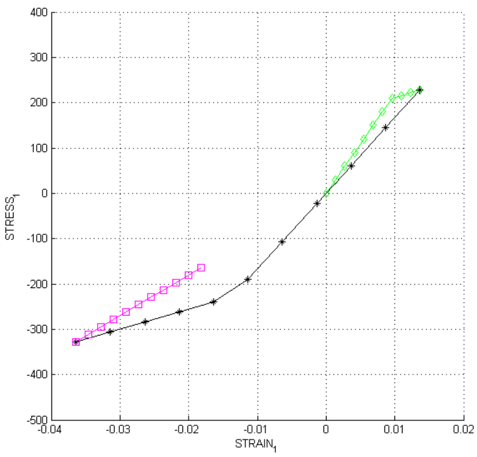


(a) Stress path without viscous effect

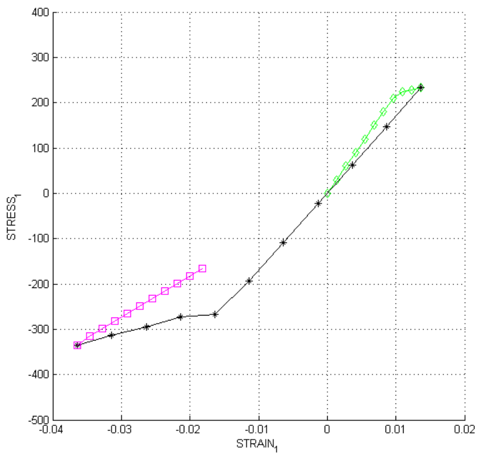


(b) Stress path with viscous effect

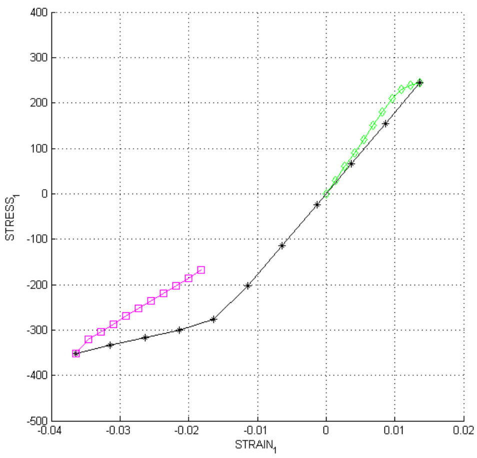
Figure 2.1: Stress path - symmetric damage model



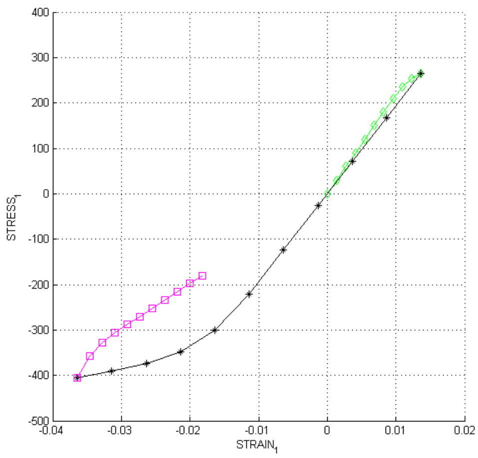
(a) Stress-strain curve without viscous effect



(b) Stress-strain curve with $\eta = 0.1$

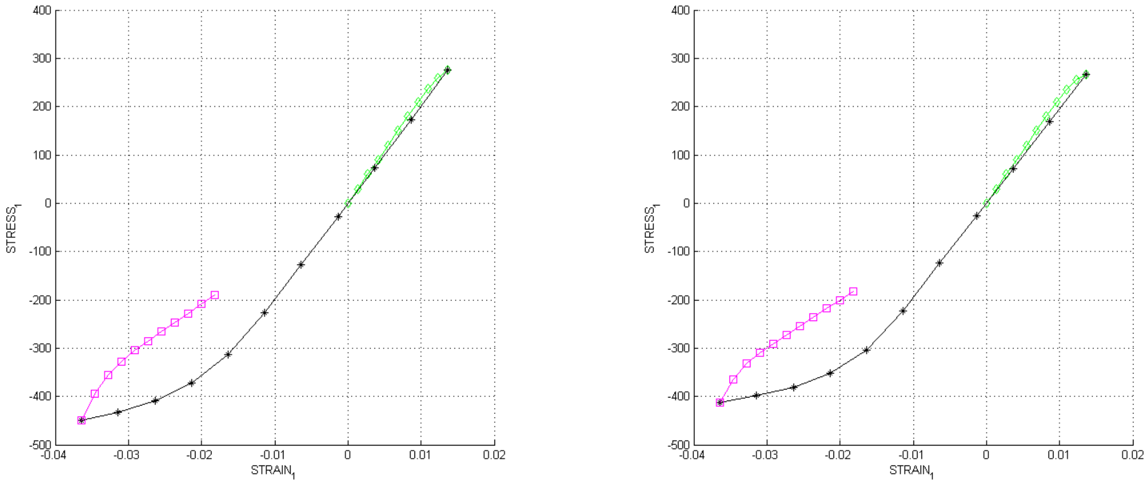


(c) Stress-strain curve with $\eta = 0.3$



(d) Stress-strain curve with $\eta = 0.9$

Figure 2.2: Effect of viscous coefficient on stress-strain curve



(a) Considering total time = 2

(b) Considering total time = 3

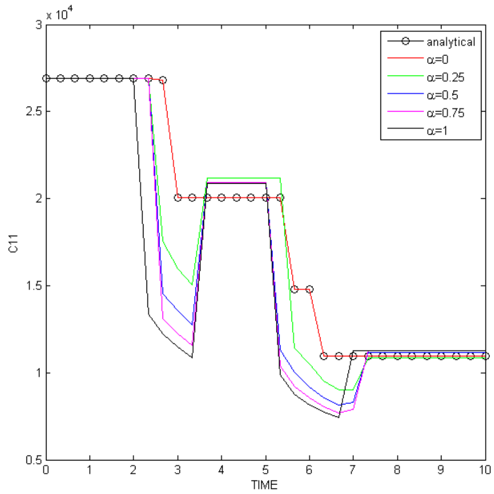
Figure 2.3: Effect of strain rate on stress-strain curve

2.2.3 Effect of integration coefficient

Performing numerical integrations by alpha method yields stable results when using integration coefficient between 0.5 to 1. Figure 2.5 shows the evolution of the results from instability to stability when integration coefficient was varied from 0 to 1. Spiky points in Figure 2.5a and Figure 2.5b disappear when larger integration coefficients were used.

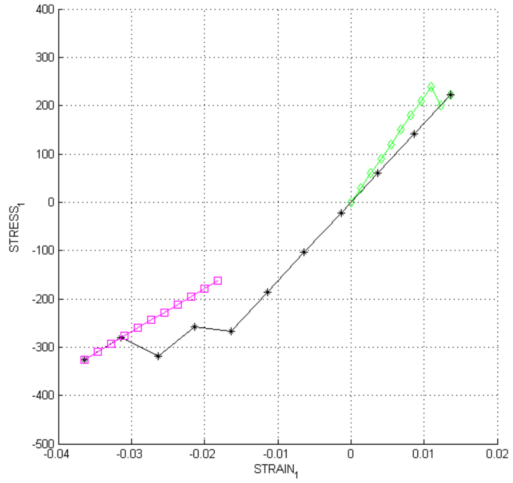
2.2.4 Tangent and algorithmic constitutive operators

It can be seen in Figure 2.4a that when $\alpha = 0$, algorithmic and analytical tangent operators match.

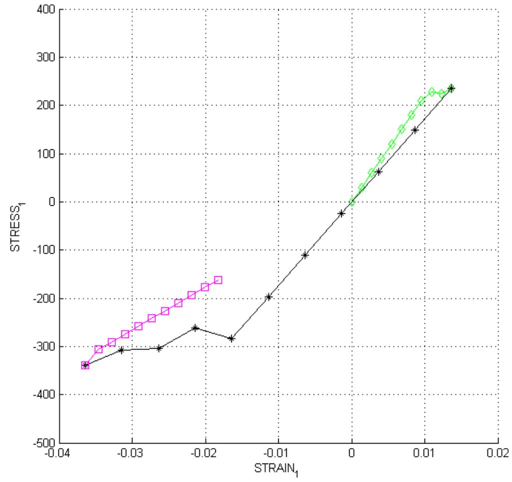


(a) effect of integration coefficient on C11

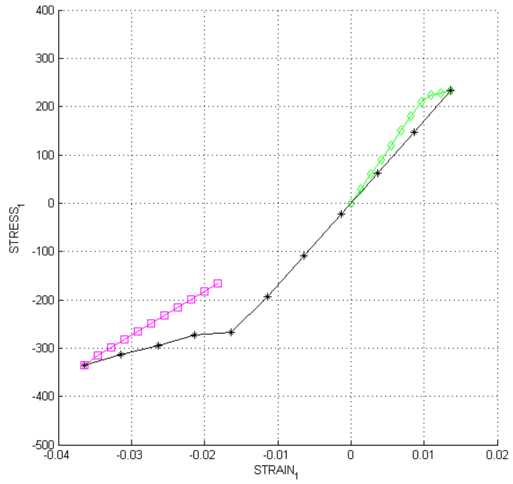
Figure 2.4: Effect of integration coefficient on stress-strain curve



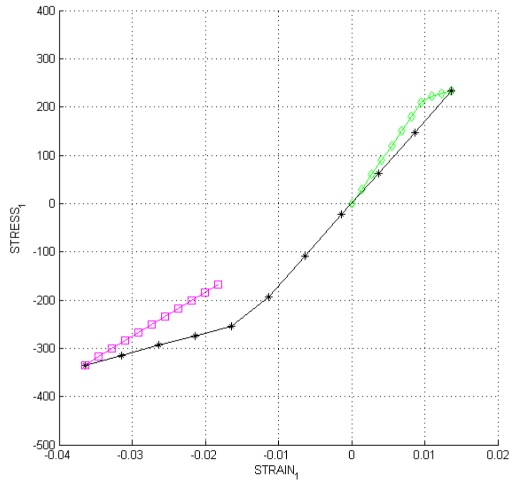
(a) Integration coefficient $\alpha = 0$



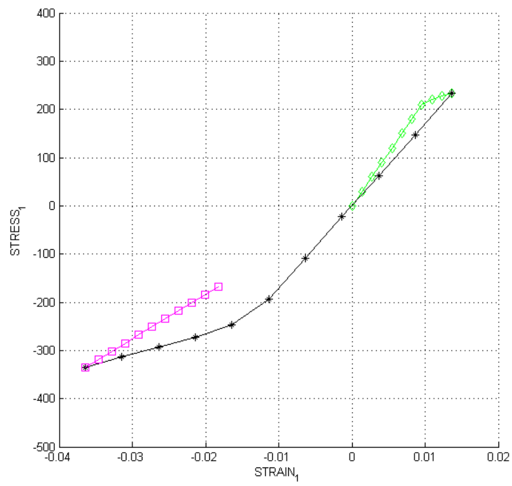
(b) Integration coefficient $\alpha = 0.25$



(c) Integration coefficient $\alpha = 0.5$



(d) Integration coefficient $\alpha = 0.75$



(e) Integration coefficient $\alpha = 1$

Figure 2.5: Effect of integration coefficient on stress-strain curve

Appendices

Modified routines

The programming codes implemented in this study are included in this appendix.

To implement tension-only damage model and non-symmetric damage model, the following routines were included in dibujar_criterio_dano1.m and Modelos_de_dano1.m

```
elseif MDtype==2 %tension-only
    tetha=[-pi/2*0.92:0.01:0.97*pi];
    %*****************************************************************************************
    %* RADIUS
    D=size(tetha);
    m1=cos(tetha); temp=m1; temp(temp<0)=0;
    m1_brac=temp;
    m2=sin(tetha); temp=m2; temp(temp<0)=0;
    m2_brac=temp;
    m3=nu*(m1+m2); temp=m3; temp(temp<0)=0;
    m3_brac=temp;

    Contador=D(1,2);
    radio = zeros(1,Contador); s1 = zeros(1,Contador); s2 = zeros(1,Contador);

    for i=1:Contador
        radio(i) = q/sqrt([m1_brac(i) m2_brac(i) 0 m3_brac(i)]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]');
        s1(i) = radio(i)*m1(i);
        s2(i) = radio(i)*m2(i);
    end

    hplot =plot(s1,s2,tipo_linea); axis equal
```

```
elseif MDtype==3 %non-symmetric
    tetha=[0:0.01:2*pi];
    %*****************************************************************************************
    %* RADIUS
    D=size(tetha);
    m1=cos(tetha); temp=m1; temp(temp<0)=0;
    m1_brac=temp;
    m2=sin(tetha); temp=m2; temp(temp<0)=0;
    m2_brac=temp;
    m3=nu*(m1+m2); temp=m3; temp(temp<0)=0;
    m3_brac=temp;

    Contador=D(1,2);
    radio = zeros(1,Contador); s1 = zeros(1,Contador); s2 = zeros(1,Contador);

    for i=1:Contador
        wt(i) = (m1_brac(i)+m2_brac(i)+m3_brac(i))/(abs(m1(i))+abs(m2(i))+abs(m3(i)));
        mod_func(i) = wt(i)+(1-wt(i))/n;
        radio(i) = q/sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]')/mod_func(i);
        s1(i) = radio(i)*m1(i);
        s2(i) = radio(i)*m2(i);
    end
```

```
elseif (MDtype==2)  %* Only tension

    temp=eps_n1;
    temp(temp<0)=0;
    eps_n1_brac=temp;
    rtrial= sqrt(eps_n1_brac*ce*eps_n1');
```

```
elseif (MDtype==3)  %*Non-symmetric

    temp=eps_n1;
    temp(temp<0)=0;
    eps_n1_brac=temp;
    wt=(eps_n1_brac(1)+eps_n1_brac(2)+eps_n1_brac(4))/(abs(eps_n1(1))+abs(eps_n1(2))+abs(eps_n1(4)));
    mod_func=wt+(1-wt)/n;
    rtrial= mod_func*sqrt(eps_n1*ce*eps_n1');

end
```

To implement exponential hardening and softening law, the following routine was included in rmap_dano1.m

```
if hard_type == 0
    %  Linear
    q_n1= q_n+ H*delta_r;
else
    % exponential
    A=abs(H);
    if H>0
        q_inf = 2*r0;
    else
        q_inf = r0/2;
    end
    q_n1 = q_inf-(q_inf-r0)*exp(A*(1-rtrial/r0));
end
```

To implement the effect of viscosity, the following routine was modified from rmap_dano1.m and included in rmap_vis_dano1.m

```
%*************************************************************************************
% Damage surface                                                                   %*
[tal_n] = Modelos_de_dano1 (MDtype,ce,eps_n,n);
[tal_n1] = Modelos_de_dano1 (MDtype,ce,eps_n1,n);
tal_na = (1-alpha)*tal_n+alpha*tal_n1;
rtrial = tal_na;


if(rtrial > r_n)
    % Loading
    fload=1;
    r_n1 = (eta-delta_t*(1-alpha))/(eta+alpha*delta_t)*r_n + delta_t/(eta+alpha*delta_t)*rtrial;
    delta_r=r_n1-r_n;

    if hard_type == 0
        %  Linear
        q_n1= q_n+ H*delta_r;
    else
        % exponential
        A=H;
        q_inf = 2*r0;
        q_n1 = q_inf-(q_inf-r0)*exp(A*(1-rtrial/r0));
    end

    if(q_n1<zero_q)
        q_n1=zero_q;
    end
else
```

```
% Damage variable
% ---------------
dano_n1   = 1.d0-(q_n1/r_n1);
%  Computing stress
%  ****************
sigma_n1  =(1.d0-dano_n1)*ce*eps_n1';

% computing constitutive operator
ce_vd = (1.d0-dano_n1)*ce + alpha*delta_t/(eta+alpha*delta_t)/tal_n1*(H*r_n1-q_n1)/r_n1^2*...
    (ce*eps_n1')*(ce*eps_n1')';
C11=ce_vd(1,1);
Ce11=(1.d0-dano_n1)*ce(1,1);
```