Assignment 2
Computational Solid Mechanics

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MSc Computational Mechanics

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1D Computational plasticity

The material data with which the numerical simulation has been carried out are considered hypothetically to be:

1) Elastic Modulus = 25 (units)
2) Yield Stress = 10 (units)

And a cyclic loading of 20,-30 and 20 (units)is considered to be the load case.
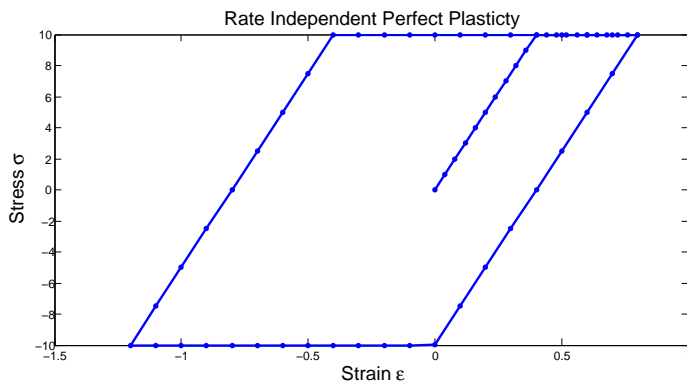
Perfect Plasticity Model

Figure 1: Rate Independent Plasticity
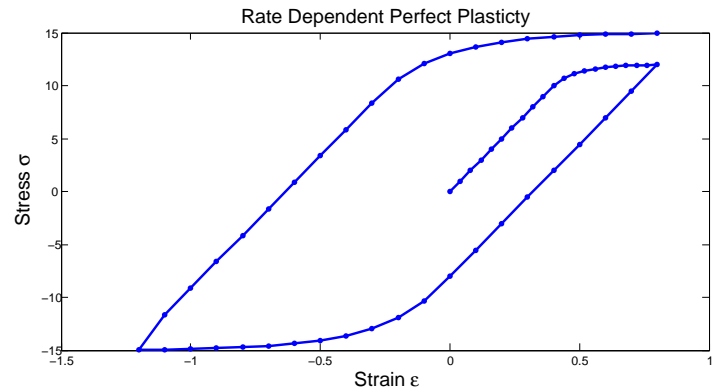

Figure 2: Rate Dependent Plasticity,$\eta = 5$

For the rate indepndent case, when cyclic loading of 20, -30 and 20 (units) is applied, it does not cross the yield surface at 10 (units) due to the absensce of any hardening parameter as seen in Figure 1.

However , in case of Rate dependent perfect plasticity, due to the presence of viscosity, when tensile loading of 20 (units) is applied, it crosses the yield surface. Further when 30 (units) compressive loading is applied, the stress go further beyond the yield surface as een in Figure. 2.
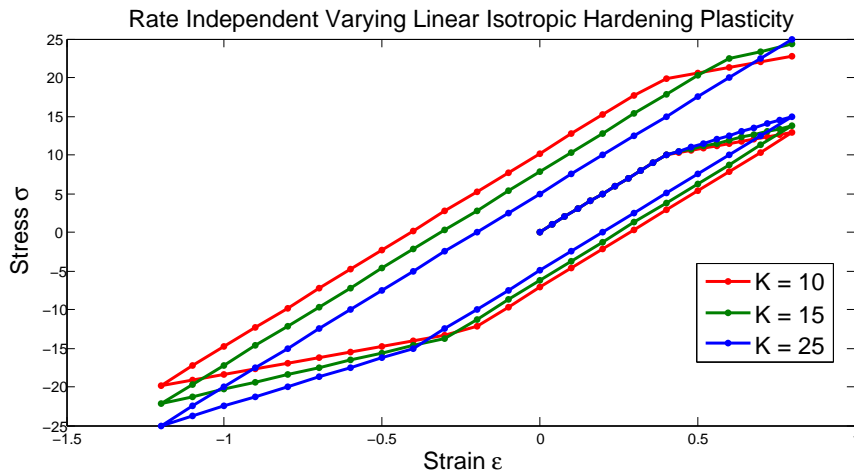
Linear Isotropic Model

Figure 3: Varying Linear Isotropic Hardening

Here a cyclic loading 20, -30, 20 (units) is applied. The yield stress during initial tension loading increases due to hardening and then the compression yield stress grows the same amount for varying Isotropic Hardening coefficient K. As we increase the value of K, the slope of the deformation changes and approaches the value of Young's Modulus (25 units) as seen in Figure. 3. The material starts behaving elastically when isotropic hardening coefficient equals young modulus .
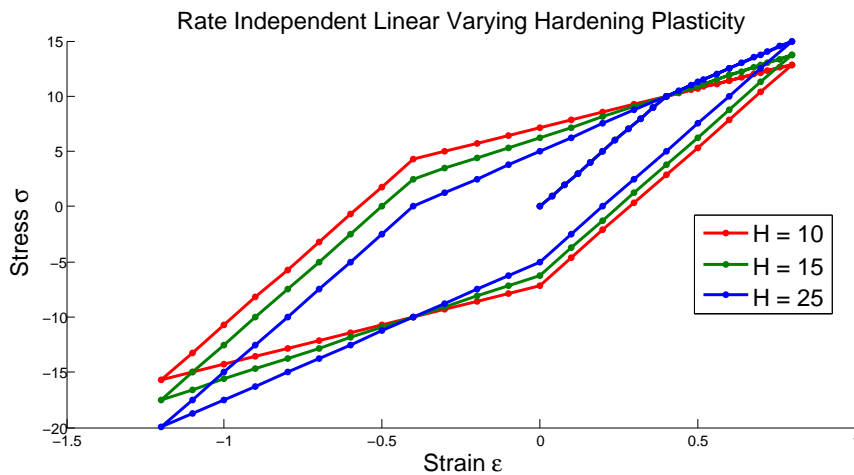
Linear Kinematic Model

Figure 4: Varying Linear Kinematic Hardening

Kinematic hardening correctly accounts for Bauschinger effect. When kinematic hardening is applied, the material softens in compression and thus can correctly model cyclic behaviour. As the Kinemtic Hardening coefficient increases, the symmetric nature of tension-compression is gradually lost as the material softens more in compression.

2

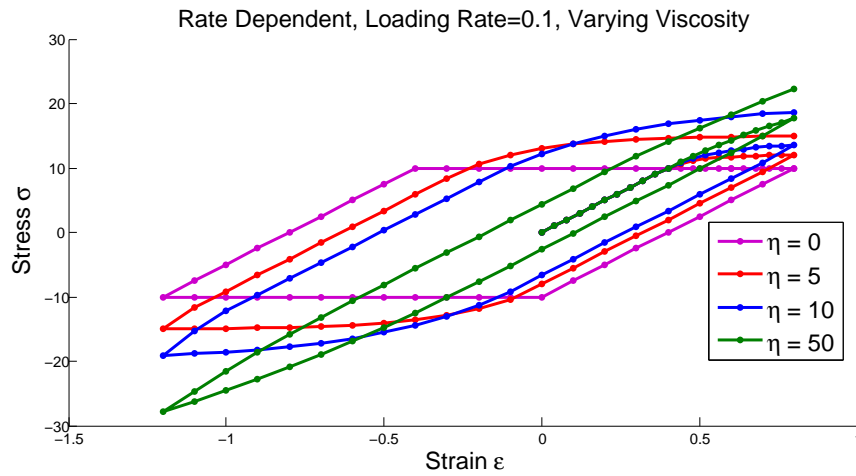# Rate Dependent Plasticty

Effect of Varying Viscosity

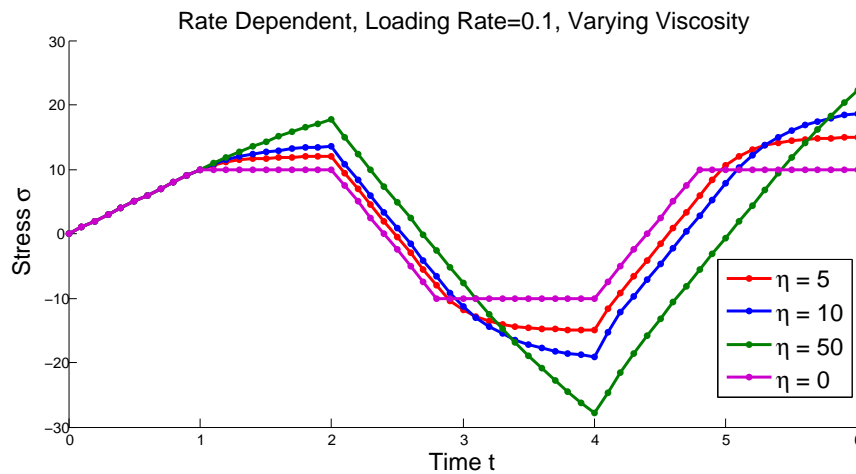Figure 5: Varying Viscosity with constant Loading Rate(Stress vs. Strain)

Figure 6: Varying Viscosity with constant Loading Rate(Stress vs.Time)

Here the loading cycle is 20,-30,20 (units). The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant strain rate and we vary the viscosity coefficients to understand the influence of viscosity parameter (η). The isotropic and kinematic hardening coefficients are taken to be zero for better understanding the effects of viscosity.

When the viscosity is zero, the material behaves like perfect plasticity condition with stresses not going beyond yield stress.As the viscosity is increased, the material gets deformed and the stress go beyond the yield stress. Upon further increase of viscosity, the streses go much further than the yield surface as seen in Figure. 6. If the coefficient of viscosity is taken to of be very high value, the

material will start behaving elastically. The plastic strain decreases with the increase of viscosity as seen in Figure 7 and 8 below.
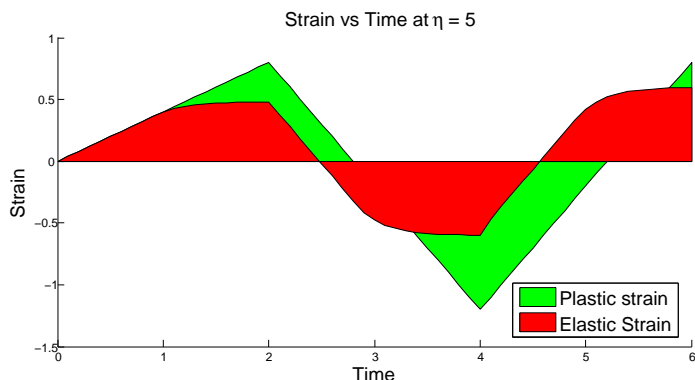

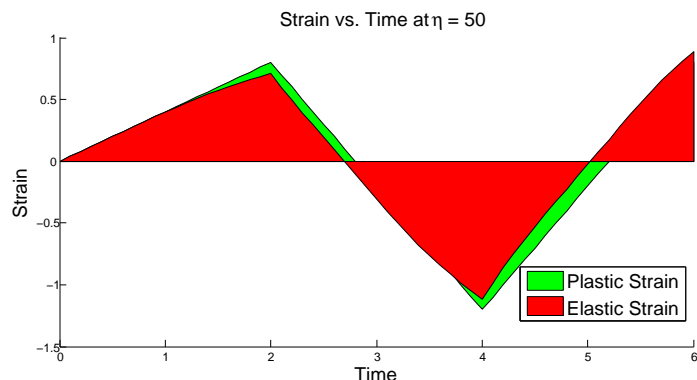Figure 7: Strain vs. Time at $\eta = 5$

Figure 8: Strain vs. Time at $\eta = 50$

Effect of Varying Loading Rate

Here the loading cycle is 20,-30,20 (units). The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant viscosity and we vary the loading rate to understand the influence of loading rate (dt). The isotropic and kinematic hardening coefficients are taken to be zero for better understanding of the effects.
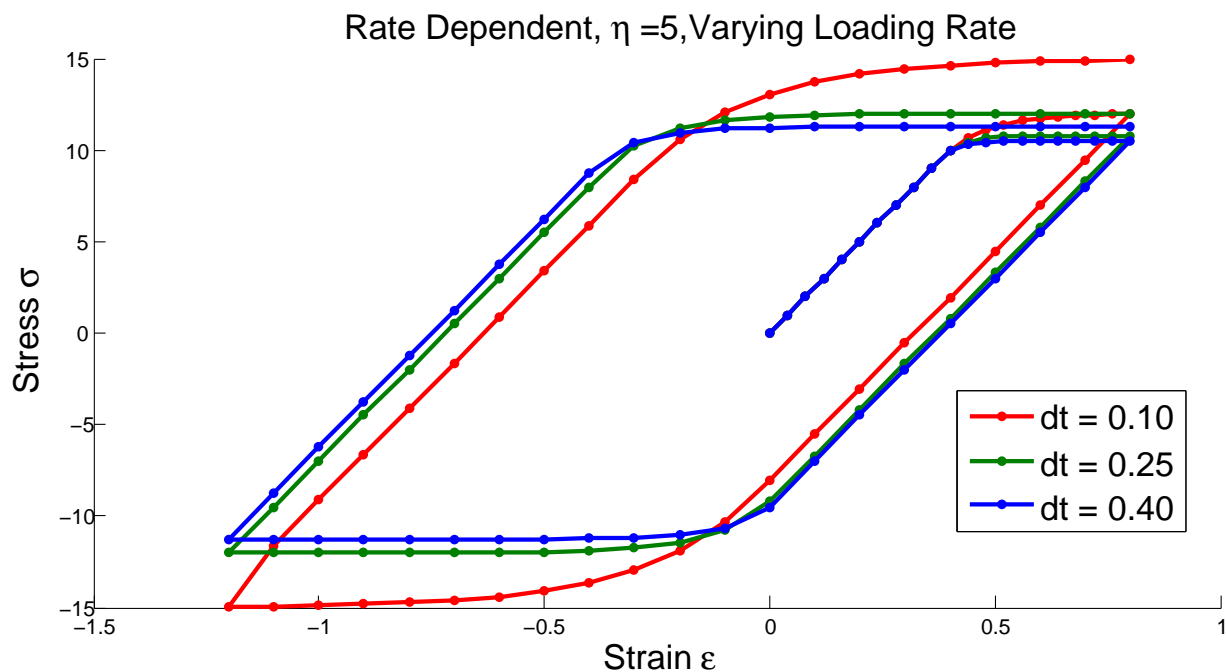

Figure 9: Varying Loading Rate with constant Viscosity(Stress vs. Strain)
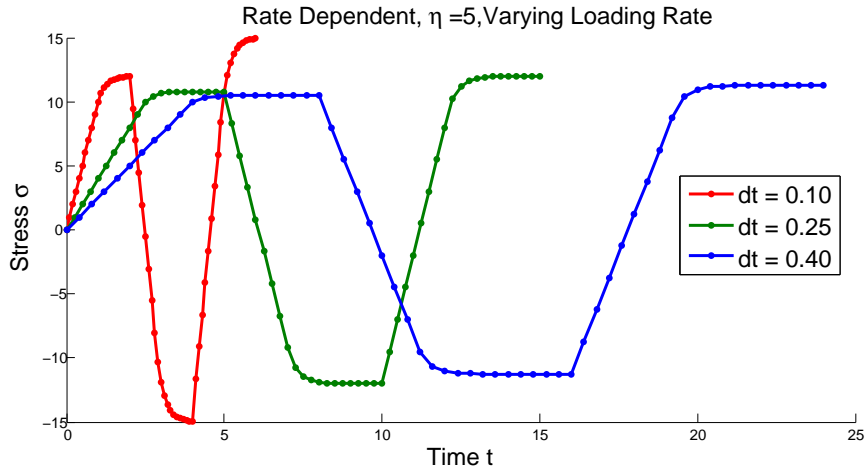
Figure 10: Varying Loading Rate with constant Viscosity(Stress vs.Time)

When dt is low, i.e. the strain rate is high, the effects of viscosity is clearly visible. The stresses exceed the yield stress as seen in Figure 9. As the strain rate is decreased, i.e. dt is increased the rate of increase of stress becomes gradually lower as seen in 10. Upon further increase of dt, i.e. when strain rate is made much lower, the material approaches quasi-static state. If the loading rate is very low , the material will start behaving perfectly plastic with little or no effect of viscosity as seen in Figure 12.

Figure 11: High Strain rate at $dt = 0.1$
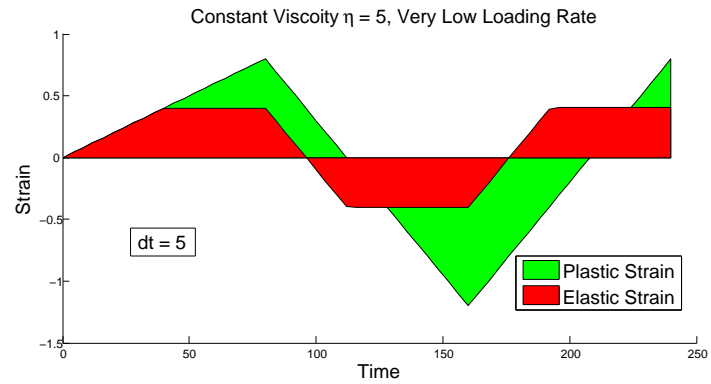
Figure 12: Very Low Strain Rate $dt = 5$

Recovery of Rate Independent response from Rate Dependent Result

Here the loading cycle is 20,-30,20 (units). The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant loading rate and we vary the viscosity to recover the rate independent response. The isotropic and kinematic hardening coefficients are taken to be zero for better understanding.

Figure 13: Varying Viscosity with constant Loading Rate(Stress vs. Strain)

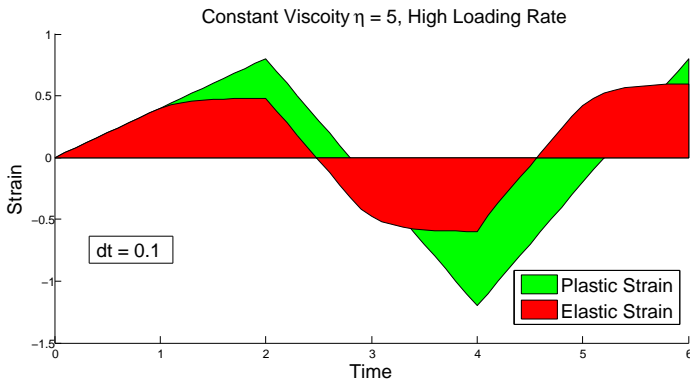Figure 14: Varying Viscosity Rate with constant Loading Rate(Stress vs. Time)

From the Figure. 13 & 14 above, we observe that, when very small value of viscosity coefficient is considered, we recover the Rate Independent response from Rate Dependent result.

6

Non-Linear Isotropic Hardening

Influence of Exponential Coefficient

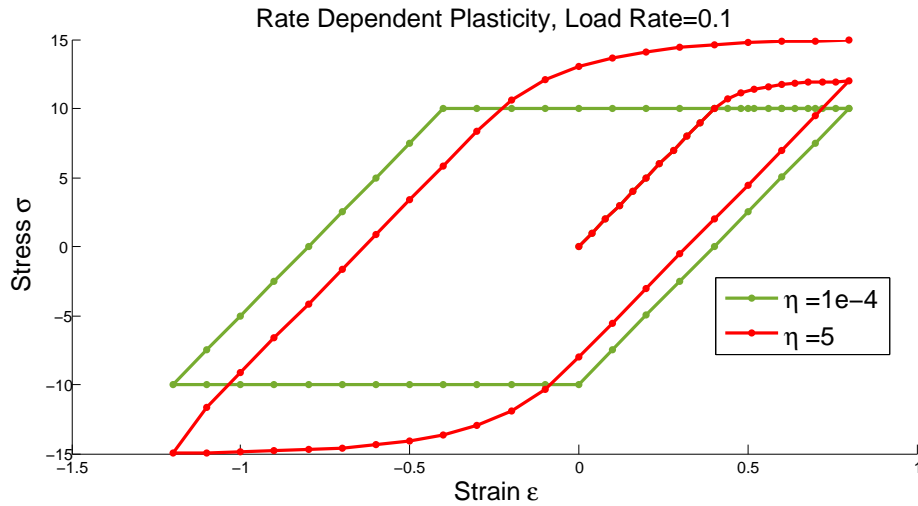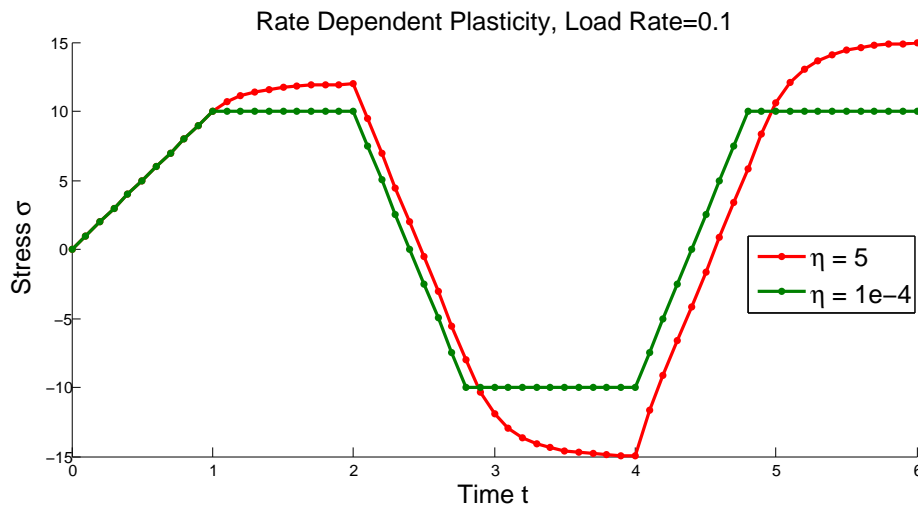Here the loading cycle is 20,-30,20 (units). The Yield stress is 10 (units), Young's modulus = 25, $\sigma_\infty = 15$ (units). We consider a rate independent non linear isotropic plasticity model for simplicity and better understanding. While calculation of non-linear isotropic hardening, the linear hardening effect is not added to the exponential saturation law, since we are considering **only the saturation law**.
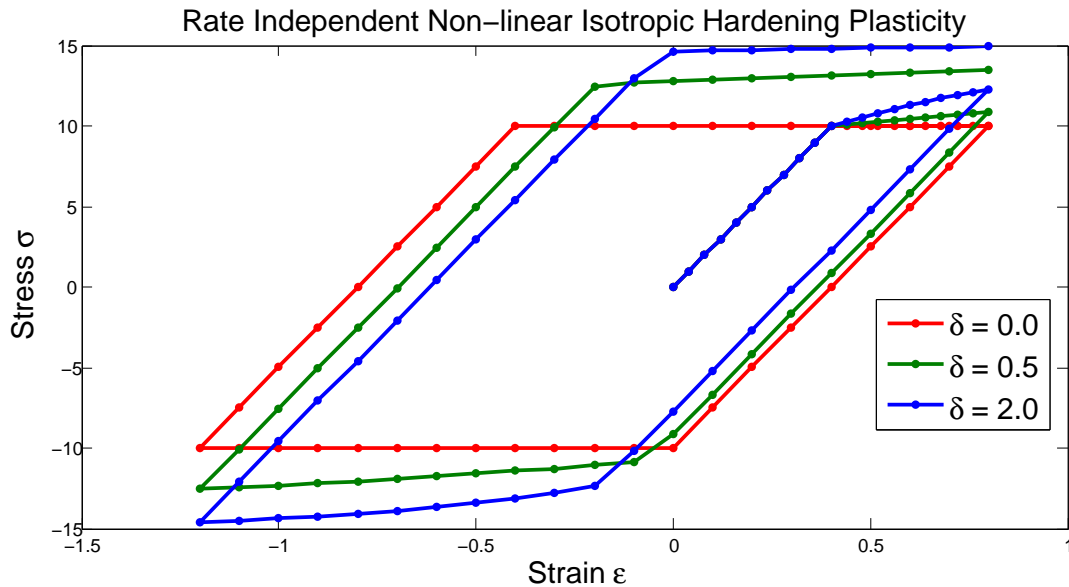
Figure 15: Rate Independent Non-linear Isotropic Hardeing Plasticity with varying delta

When the exponential coefficient δ is zero, the material gives perfect plastic response. As the value of δ is increased we start to get a non-linear exponential hardening plot which approaches the value of σ_∞. The stress will never go beyond the saturation point. With increasing value of δ, the tangential slope of hardening, approaching σ_∞ increases.

We next consider a Rate Dependent Non-Linear Isotropic and Linear Kinematic Hardening case. Here the loading cycle is 20,-30,20 (units). The Yield stress is 10 (units), Young's modulus = 25, σ_∞ = 15 (units) , viscosity $\eta = 5$, $\delta = 1$, Kinematic Hardening = 10 (units). We get the following plots.
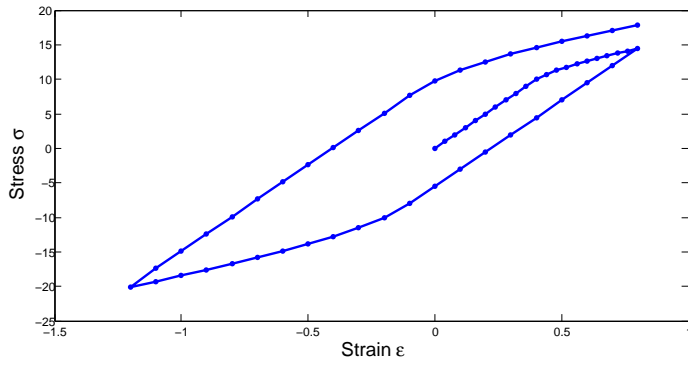
Figure 16: Stress vs. Strain
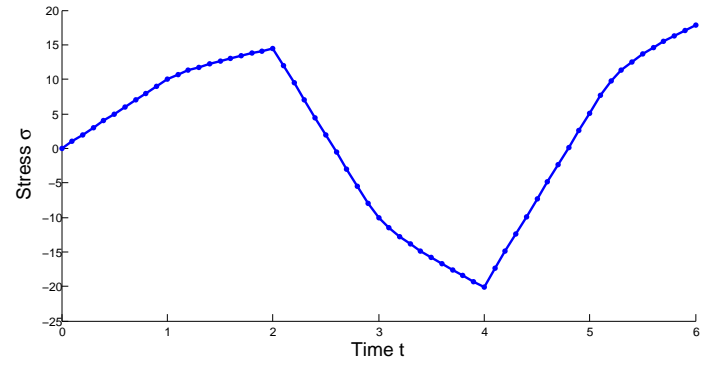
Figure 17: Stress vs. Time
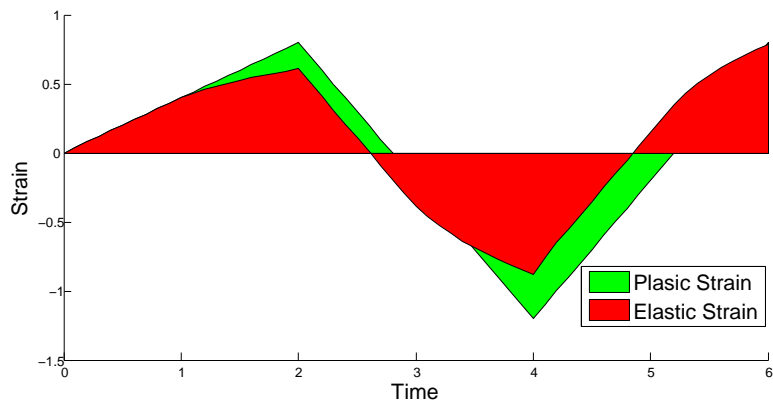
Figure 18: Strain vs. Time

APPENDIX

```matlab
1  clear all
2  clc
3
4  %%--------------        Material Properties---------------
5  E          =   25;      %Young modulus
6  yield      =   10;      %Yield stress
7  eta        =   5;       %Viscosity
8  K          =   10;       %Isotropic Hardening
9  H          =   10;       %Kinematic Hardening
10 sigma_infy =   15;
11 %%----------------------------------------------------
12 delta=1;
13 %--------------        Loading Rate--------------------
14 dt=0.1;
15
16 disp('[1] Linear case')
17 disp('[2] Nonlinear case')
18 method=input('Choose method:');
19
20 %%--------------        Loading      --------------
21 nSteps          =   20;
22 sigmaP          =   [20;-30;20];
23 load_Steps      =   size(sigmaP,1) ;
24 timeSteps       =   nSteps*ones(1,load_Steps);
25 %%----------------------------------------------------
26
27 strain_History  =   StrainHistory(E,sigmaP,timeSteps);
28
29 if method==1        %Linear Case
30
31 sigma_trail     =   zeros(1 , length(strain_History)) ;
32 f_trial         =   zeros(1 , length(strain_History)) ;
33 zeta_trial      =   zeros(1 , length(strain_History)) ;
34 stress          =   zeros(1 , length(strain_History)) ;
35 strain_plas     =   zeros(1 , length(strain_History)) ;
36 qbar            =   zeros(1 , length(strain_History)) ;
37 alpha           =   zeros(1 , length(strain_History)) ;
38
39
40 for n=1 : length(strain_History)-1
41     sigma_trail(n+1)    = E * (strain_History(n+1) - strain_plas(n)) ;
42     zeta_trial(n+1)     = sigma_trail(n+1) - qbar(n) ;
43     f_trial(n+1)        = abs(zeta_trial(n+1)) - ( yield + K * alpha(n) ) ;
44
45     if f_trial(n+1) <= 0    %% within elastic limit
46
47         stress(n+1)     =  sigma_trail(n+1) ;
48         strain_plas(n+1) =  strain_plas(n);
49         qbar(n+1)       =  qbar(n);
50         alpha(n+1)      =  alpha(n) ;
51
52     else                    %% Plasicity starts
53         %Updation of internal variables takes place
54
55         gamma           = ramp_fn(f_trial(n+1)) / (E + K + H + eta / dt) ;
56         stress(n+1)     = sigma_trail(n+1) - gamma * E ...
57                                             *sign(zeta_trial(n+1)) ;
58
59         strain_plas(n+1) = strain_plas(n) + gamma ...
60                                             * sign(zeta_trial(n+1)) ;
```

```matlab
61
62          alpha(n+1)          = alpha(n) + gamma ;
63          qbar(n+1)           = qbar(n) + gamma * H * sign(zeta_trial(n+1)) ;
64
65      end
66
67 end
68
69 elseif method==2     %Non Linear
70
71 dt=1e-1;
72 sigma_trail      =     zeros(1 , length(strain_History)) ;
73 f_trial          =     zeros(1 , length(strain_History)) ;
74 zeta_trial       =     zeros(1 , length(strain_History)) ;
75 stress           =     zeros(1 , length(strain_History)) ;
76 strain_plas      =     zeros(1 , length(strain_History)) ;
77 qbar             =     zeros(1 , length(strain_History)) ;
78 q                =     zeros(1 , length(strain_History)) ;
79 alpha            =     zeros(1 , length(strain_History)) ;
80 gamma            =     zeros(1 , length(strain_History)) ;
81
82 for n=1 : length(strain_History)-1
83
84      sigma_trail(n+1)    =    E * (strain_History(n+1) - strain_plas(n)) ;
85      zeta_trial(n+1)     =    sigma_trail(n+1) - qbar(n) ;
86      q(n)                =    - phi(alpha(n),delta,sigma_infy,yield) ;
87      f_trial(n+1)        =    abs(zeta_trial(n+1)) - yield + q(n) ;
88
89      if f_trial(n+1) <= 0        %Elastic state
90
91          stress(n+1)         =    sigma_trail(n+1) ;
92          strain_plas(n+1)    =    strain_plas(n);
93          qbar(n+1)           =    qbar(n);
94          alpha(n+1)          =    alpha(n) ;
95
96      else                        %plastic state
97          %Updation of internal variables takes place
98
99          gamma(n+1) = Newton(f_trial(n+1),dt,E,H,eta,alpha(n),delta...
100                                             ,sigma_infy,yield) ;
101
102          stress(n+1) = sigma_trail(n+1) - gamma(n+1)*dt * E ...
103                                             *sign(zeta_trial(n+1)) ;
104
105          strain_plas(n+1) = strain_plas(n) + gamma(n+1)*dt  ...
106                                             *sign(zeta_trial(n+1)) ;
107
108          alpha(n+1) = alpha(n) + gamma(n+1)*dt ;
109          qbar(n+1)  = qbar(n) + gamma(n+1)*dt * H * sign(zeta_trial(n+1)) ;
110
111      end
112 end
113
114 end
115
116 %--------------------          Postprocssing-----------------------
117 hold on
118 time=0:dt:(dt*nSteps*load_Steps);
119
120  figure(1)
```

```matlab
121 plot(time,stress,'.-','LineWidth',2,'Markersize',15)
122 title('Rate Dependent, \eta =5,Varying Loading Rate dt','Fontsize',16)
123 xlabel('Time t','Fontsize',16)
124 ylabel('Stress \sigma','Fontsize',16)
125
126 figure(2)
127 plot(strain_History,stress,'.-','LineWidth',2,'Markersize',15)
128 title('Rate Dependent, \eta =5,Varying Loading Rate dt','Fontsize',16)
129 xlabel('Strain \epsilon','Fontsize',16)
130 ylabel('Stress \sigma','Fontsize',16)
131
132 figure(3)
133 area(time,(strain_History),'facecolor','g')
134 area(time,(strain_History-strain_plas'),'facecolor','r')
135
136 %-----------------------------------------------------------
137
```

```matlab
1 function gamma2=Newton(trial_f,dt,E,H,eta,alpha,delta,sigma_infy,yield)
2 gamma1=0;
3 relErr=1;
4
5 while (relErr>1e-13)
6
7     gamma2=gamma1-residue(gamma1,trial_f,dt,E,H,eta,alpha,delta...
8                     ,sigma_infy,yield)/D_residue(gamma1,dt,E,H...
9                            ,eta,alpha,delta,sigma_infy,yield);
10
11
12     relErr=abs(gamma2-gamma1);
13     gamma1=gamma2;
14
15
16 end
17
18 end
```

```matlab
1 function value=residue(gamma,trial_f,dt,E,H,eta,alpha,delta,sigma_infy,yield)
2
3 value = trial_f - gamma * dt * (E+H+eta/dt) -...
4                 (phi(alpha + gamma * dt,delta,sigma_infy,yield) -...
5                            phi(alpha,delta,sigma_infy,yield));
6 end
```

```matlab
1 function value = D_residue(gamma,dt,E,H,eta,alpha,delta,sigma_infy,yield)
2
3 value =  - dt * (E + (sigma_infy - yield) * delta *...
4             exp(- delta * (alpha + gamma * dt)) + H + eta / dt);
5
6 end
```

```matlab
1 function value=phi(alpha,delta,sigma_infy,yield)
2
3 value = (sigma_infy - yield)*(1 - exp(-delta*alpha)) ;
4
5 end
```

```matlab
 1 function strain=StrainHistory(E,sigma,istep)
 2
 3 sigma=[0;sigma];
 4 STRAIN = zeros(size(sigma,1),1);
 5
 6 for   iloc = 1:size(sigma,1)-1
 7
 8     sigma_0 =sigma(iloc+1,1);
 9     strain_di=sigma_0/E;
10     STRAIN(iloc+1,1)=strain_di;
11 end
12
13 [strain] = calstrain_IN(istep,STRAIN);
14 end
```

```matlab
1 function ramp = ramp_fn(value)
2
3 ramp = ( abs(value) + value ) / 2;
4
5 end
```