# **Computational Solid Mechanics**

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# **MSc Computational Mechanics**

## **Assignment 1**

# **April 7, 2017**

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The integration algorithm is applied for non-linear tension-compression and tension only damage model in the supplied code and can be seen in annexure. And all the cases are solved using yield stress = 200.

Linear and hardening cases are applied and the code can be seen in annexure, all the cases are solved using HARDSOFT-MOD=-0.1(for softening) and HARDSOFT-MOD=0.1(for hardening)

3.1. Tension only damage model

#### 3.1.1. Linear

For all three cases  $\alpha = 200$ ,  $\beta = 300$ ,  $\gamma = 250$ 

In figure(1) we can see that first load is applied up to 200 and then compression loading is applied is up to point 9 then again tensile loading is applied which exceeds the yield stress, so the surface is damaged and also one damaged surface is seen as loading exceeds only once the yield stress at point 13.

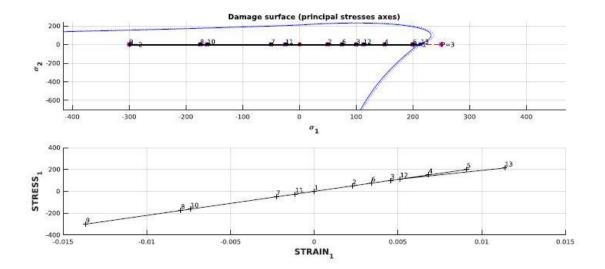


Figure 1: Case 1 tension only, linear law

In case 2 as shown in figure(2) first uniaxial tensile loading is applied so the load line follows x-axis till P=1, then biaxial tensile unloading is applied afterwards biaxial tensile loading is applied and the surface is damaged.

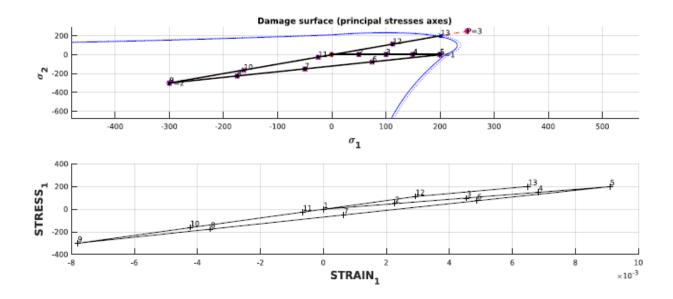


Figure 2: Case 2 tension only, linear law

In case 3 biaxial tensile loading is applied first followed by biaxial tensile loading then biaxial tensile loading is applied because of which the load line follows a similar pattern while unloading and then loading upto 170(apprx.) then it follows a different pattern and damage the surface, as its more than that yield stress and the same phenomenon can be seen in figure(3). Also point P=3 shows that where will be damaged surface if the material is elastic.

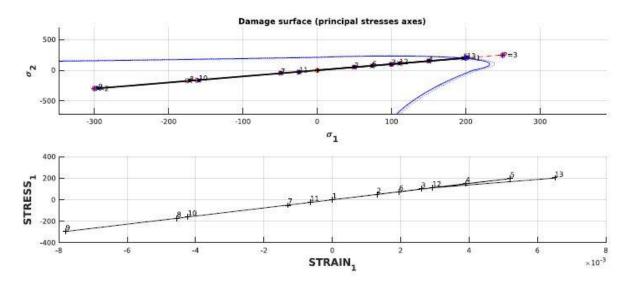


Figure 3: Case 3 tension only, linear law

#### 3.1.2 Exponential

For all three cases  $\alpha = 400$ ,  $\beta = 600$ ,  $\gamma = 300$ .

Now using exponential hardening higher loads are applied so number of damaged surfaces will be more as the load will we go above yield stress multiple times. For case one the first uniaxial tensile loading will exceed the yields stress, hence one damage surface is created then uniaxial tensile unloading is done after that uniaxial tensile loading also exceeds the yield stress so second damaged surface is created as shown in figure(4). And also in the stress vs strain plot it can be seen that loading and unloading follows a same path up to point 0.

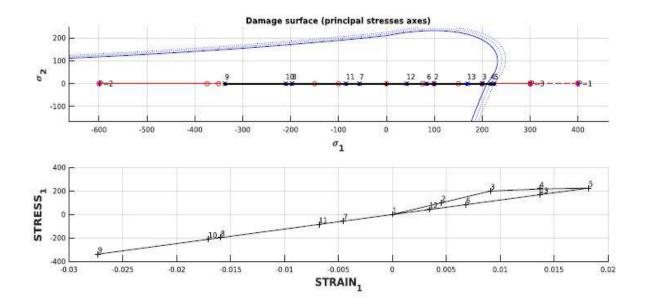


Figure 4: Case 1 tension only, exponential law

In case 2 figure(5), uniaxial tensile loading is applied and at point 3 it crosses the yield stress and enters partial plasticity zone then biaxial tensile unloading is applied and it goes back to elastic region then biaxial tensile loading is applied but it does not crosses the yield stress. But still two damage surfaces are created ,because two points 4 and 5 while uniaxial loading were above yield stress.

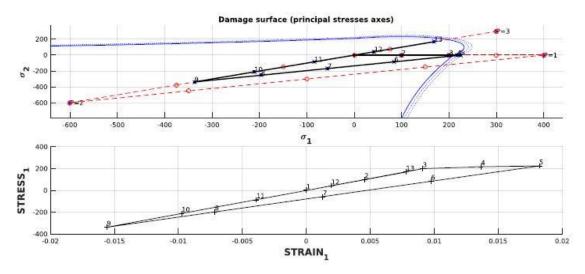


Figure 5: Case 2 tension only, exponential law

In case 3 figure(6) first a biaxial tensile loading is applied due to which it crosses the yield surface and two damage surfaces are created ,then biaxial tensile unloading is applied and then biaxial loading exactly follows the path of unloading as the body is already damaged.

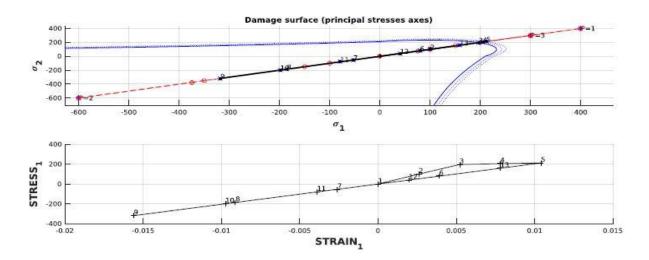


Figure 6: Case 3 tension only, exponential law

#### 3.2 Non-symmetric tension-compression damage model

#### 3.2.1. Linear

For case 1 of non-symmetric linear parameters used are  $\alpha = 300$ ,  $\beta = -550$ ,  $\gamma = 350$ .

In non-symmetric cases both tension and compression loads are applied abd the undamaged and damaged surfaces are closed surfaces. In case 1 figure(7), first a uniaxial tensile load is applied and the load crosses the yield stress at two instances i.e. at points 4 and 5. Then biaxial compressive loading is applied and biaxial tensile loading is applied and in this model it crosses the yield surface and damage so a third damaged surface can be seen.

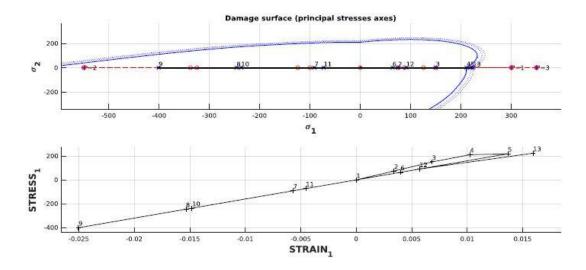


Figure 7: Case 1 Non-symmetric tension-compression, linear law

For case 2 and 3 of non-symmetric linear For all three cases  $\alpha = 300$ ,  $\beta = -550$ ,  $\gamma = 400$ . In case 2 figure(8) we can see that during the axial tensile loading the stress strain curve is increasing gradually and again the yield stress is crossed at points 4 and 5, also during the compressive loading it is occuring gradually ,but during biaxial tensile loading some distortion is there at point 12 and again yield stress is crossed and third damaged surface is formed.

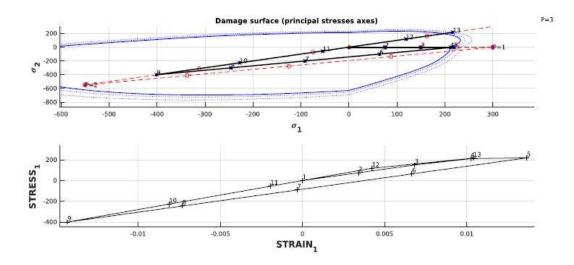


Figure 8: Case 2 Non-symmetric tension-compression, linear law

In case 3 figure(9) the uniaxial tensile loading is not gradual increase and damaged surface is formed at points 4 and 5.But biaxial compressive loading and biaxial tensile loading follows same path from point 9 to 0,0 stress strain.

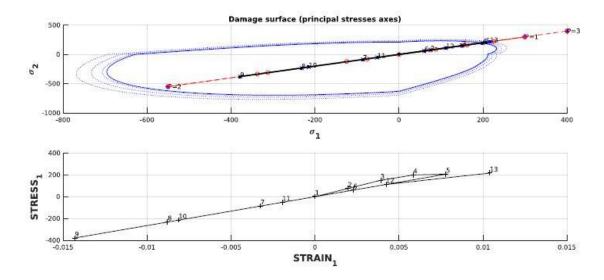


Figure 9: Case 3 Non-symmetric tension-compression, linear law

#### 3.2.2 Exponential

For case 1 following is the model:-

In case 1 figure (10) during uniaxial loading two damaged surface are created then uniaxial compressive loading is done up to point 9 and uniaxial tensile loading follows the same path as that of uniaxial compressive loading .

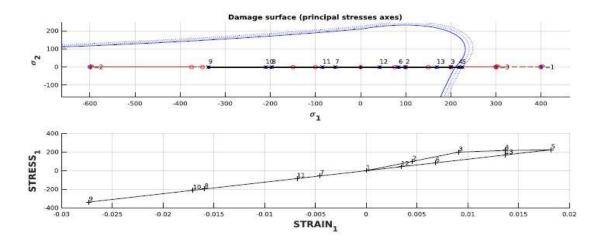


Figure 10: Case 1 Non-symmetric tension-compression, exponential law

For case 2 following is the model:-

$$\Delta \overline{\sigma}_{1}^{(1)} = 400;$$
 $\Delta \overline{\sigma}_{2}^{(1)} = 0$ 
 $\Delta \overline{\sigma}_{1}^{(2)} = -580;$ 
 $\Delta \overline{\sigma}_{2}^{(2)} = -580;$ 
 $\Delta \overline{\sigma}_{2}^{(2)} = 290;$ 
 $\Delta \overline{\sigma}_{2}^{(3)} = 290$ 

In case 2 figure(11) uniaxial tensile loading follows the same path as the loading is same as in the previous case ,but here biaxial compressive loading and biaxial tensile loading is applied and biaxial tensile loading follows the path of uniaxial tensile loading after (0,0) ,but its unavailable to reach the point of yield stress, so only two damaged surface are there in this and also the deformation of surface would ahve been very large if the case was elastic, as shown by points P=1 and P=3.

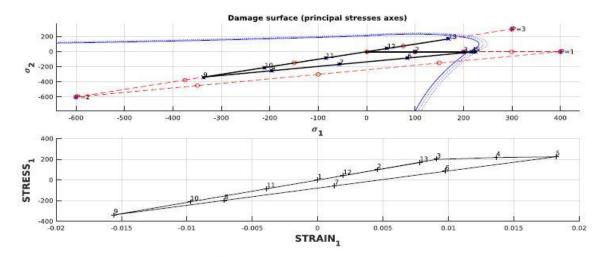


Figure 11: Case 2 Non-symmetric tension-compression, exponential law

For case 3 following is the model:-

$$\Delta \overline{\sigma}_{1}^{(1)} = 500; \qquad \Delta \overline{\sigma}_{2}^{(1)} = 500 
\Delta \overline{\sigma}_{1}^{(2)} = -750; \qquad \Delta \overline{\sigma}_{2}^{(2)} = -750 
\Delta \overline{\sigma}_{1}^{(2)} = 400; \qquad \Delta \overline{\sigma}_{2}^{(3)} = 400$$

In case 3 figure(12) biaxial tensile loading is applied ,so we can see that now while tensile loading there are three deformed surface ,while one deformed surface is very near to the undeformed surface, biaxial compressive loading follows the same pattern but biaxial tensile loading can't cross the yield stress even when it's twice the yield stress.

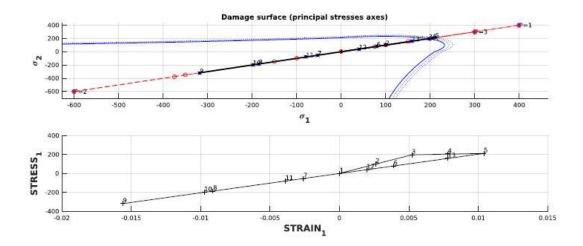


Figure 12: Case 3 Non-symmetric tension-compression, exponential law

### 3.3 Hardening/Softening

For hardening and softening in figure 13 and 14 following model is used:-

$$\begin{array}{llll} \Delta \overline{\sigma}_{1}^{(1)} = 400; & \Delta \overline{\sigma}_{2}^{(1)} = 0; \\ \Delta \overline{\sigma}_{1}^{(2)} = -580; & \Delta \overline{\sigma}_{2}^{(2)} = -580; \\ \Delta \overline{\sigma}_{1}^{(2)} = 290; & \Delta \overline{\sigma}_{2}^{(3)} = 290; \end{array}$$

This model is working as expected for hardening and softening damage. It is case 2 with exponential law. The code is working good with hardening and softening problems as the plot of r vs q in both the cases is as expected.

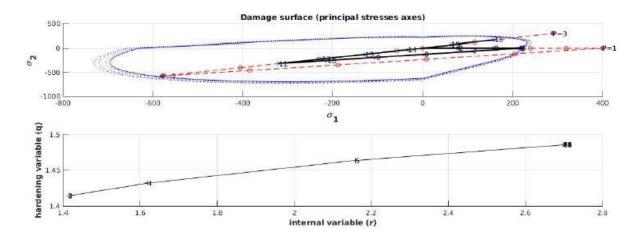


Figure 13: Hardening

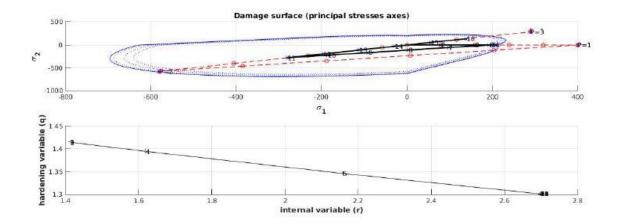


Figure 14: Softening

For hardening and softening in figure 15 and 16 following model is used:-

$$\Delta \overline{\sigma}_{1}^{(1)} = 300; \qquad \Delta \overline{\sigma}_{2}^{(1)} = 0$$

$$\Delta \overline{\sigma}_{1}^{(1)} = 300;$$
 $\Delta \overline{\sigma}_{2}^{(1)} = 0$ 
 $\Delta \overline{\sigma}_{1}^{(2)} = -550;$ 
 $\Delta \overline{\sigma}_{2}^{(2)} = -550$ 

$$\Delta \overline{\sigma}_1^{(2)}$$
=400;  $\Delta \overline{\sigma}_2^{(3)}$ =400  
Following model is also working with case 2 exponential law .

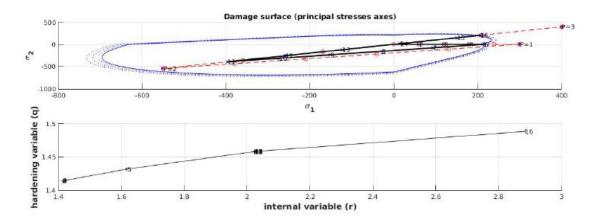


Figure 15: Hardening

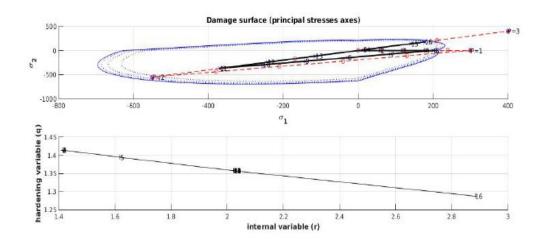


Figure 16: Softening

#### 4 Viscous

### 4.1 Variation of viscosity parameter

Figure(17) shows the damage surface and stress strain plot of a viscous linear case with  $\alpha$ = 300 ,  $\beta$  = 600  $\gamma$  = 900 loading parameters. Figure(18) is a plot of variation of viscosity parameter  $\eta$ . In this case total loading time is fixed to 100 and alpha coefficient is fixed to 0.5.

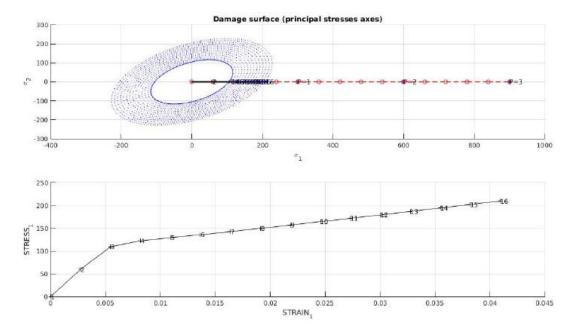
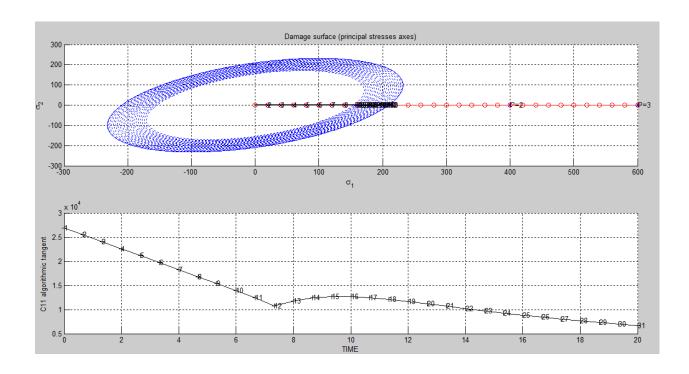


Figure 17: variation of viscosity parameter



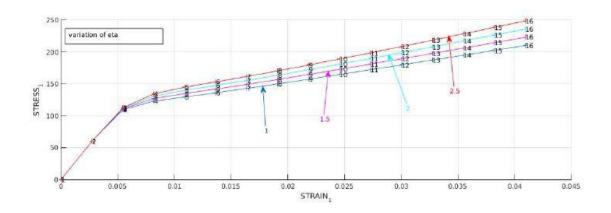


Figure 18: variation of viscosity parameter

### 4.2 Variation of strain rate

Figure (19) shows variation of strain rate is calculated by keeping  $\eta = 1$  and  $\alpha = .5$ 

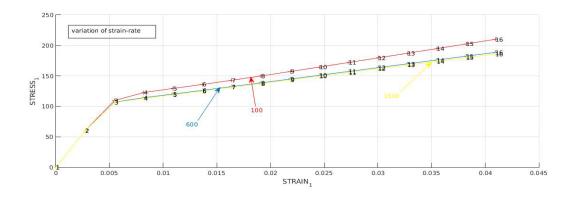


Figure 19: Variation of strain rate

### 4.3 Variation of alpha

Figure (20) variation of  $\alpha$  is calculated by putting the given values of alpha and fixing  $\eta=1$ . The plot of  $\alpha=0$  is not viscous its inviscid, so this is where the code fails. Because in a viscous case the plot of  $\alpha=0$  should be a straight line.

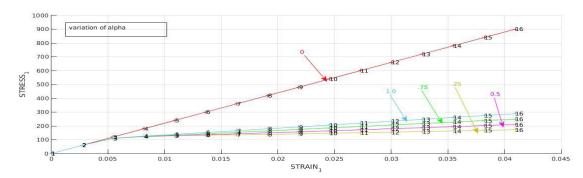


Figure 20: variation of alpha

#### **ANNEXURE**

Modelos de dano1.m

```
elseif (MDtype==2) %* Only tension
    sigma_bar = ce*eps_n1';
    sigma_p= (sigma_bar + abs(sigma_bar))/2;
    rtrial= sqrt(eps_n1*sigma_p);
    %size(eps_n1*sigma_p)
    %size(sigma_p)

elseif (MDtype==3) %*Non-symmetric
    sigma_bar = ce*eps_n1';
        sigma_p= (sigma_bar + abs(sigma_bar))/2;
    Q = sum(diag(sigma_p))/sum(diag(abs(sigma_bar)));
    rtrial= (Q + (1-Q)/n)*sqrt(eps_n1*sigma_bar);
    % size(rtrial)
    % size(sigma_bar)
    % size(sigma_bar)
    % size(sigma_bar)
```

Rmap-dano1.m for delta-t

```
function [sigma_nl,hvar_nl,aux_var] = rmap_danol (eps_nl,hvar_n,Eprop,ce,MDtype,n,delta_t)
```

Rmap-dano1.m for variables

Rmap-dano1.m for hardening variables

```
if(rtrial > r_n)
   %* Loading
   fload=1;
   delta_r=rtrial-r_n;
   r_n1= rtrial ;
   if hard type == 0
       % Linear
       q_n1= q_n+ H*delta_r;
   else
       A=H;
       q_inf=zero_q;
       q_n1=q_inf-(q_inf-r0)*exp(A*(1-(rtrial)/(r0)));
       H_n1 = A*((q_inf-r0)/r0)*exp(A*(1-rtrial/r0));
   end
   if(q_n1<zero_q)
       q_n1=zero_q;
   end
```

#### Rmap-dano1.m for viscous

```
rtrial1=rtrial;
   rtrial=(1-alpha)*r_n+alpha*rtrial1;
if(rtrial > r_n)
  %* Loading
   fload=1;
   delta_r=rtrial-r_n;
   r_n1= ((eta-delta_t*(1-alpha))*r_n)/(eta+alpha*delta_t)+((rtrial*delta_t)/(eta+alpha*delta_t)) ;
   if hard_type == 0
       % Linear
       q_n1= q_n+ H*delta_r;
       H_n1=H;
   else %exponential
       A=H;
       q_inf=zero_q;
       q_n1=q_inf-(q_inf-r0)*exp(A*(1-rtrial/r0));
       H_n1 = A*((q_inf-r0)/r0)*exp(A*(1-rtrial/r0));
   if(q_n1<zero_q)</pre>
       q n1=zero_q;
       H_n1=0;
   end
```

### Dibujar-criterio-dano1.m for MDtype2

### Dibujar-criterio-dano1.m for MDtype3

```
% Comment/delete lines below once you have implemented this case
       menu({'Damage surface "NON-SYMMETRIC" has not been implemented yet. '; ...
            'Modify files 'Modelos_de_danol' and "dibujar_criterio_danol"' ; ...
'to include this option'}, ...
       error('OPTION NOT AVAILABLE')
tetha=[0:0.01:2*pi];
                        ************************
     %* RADIUS
    D=size(tetha);
                                                    %* Range
    ml=cos(tetha);
     m2=sin(tetha);
                                                    %*
     Contador=D(1,2);
     radio = zeros(1,Contador) ;
    s1 = zeros(1,Contador);
s2 = zeros(1,Contador);
     for i=1:Contador
    for i=1:(ontagor
    sT=[ml(i) m2(i) 0 nu*(ml(i)+m2(i))]';
    sigma_plus=(sT+abs(sT))/2;
    Q=sum(sigma_plus)/(sum(abs(sT)));
radio(i)=q/((Q+(1-Q)/n)*sqrt(sT'*ce_inx*sT));
    sl(i)=radio(i)*ml(i);
          s2(i)=radio(i)*m2(i);
     hplot =plot(s1,s2,tipo_linea);
     %size(radio)
```