Assignment 1 - COSM

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Introduction

Continuum damage models have been widely accepted for simulating the behavior of materials whose mechanical properties are degrading due to the presence of small cracks that propagate during loading.

Part I (Rate Independent Models)

The "Non-Symmetric Tension-Compression Damage" model

The Figure below shows implementation of non-symmetric tension-compression damage" model which is useful to simlate the behavior of concrete, rocks and other granual materials. The code is available in Appendix A. In this kind of damage model the elastic limits of the elastic domain are different for tension and compression.

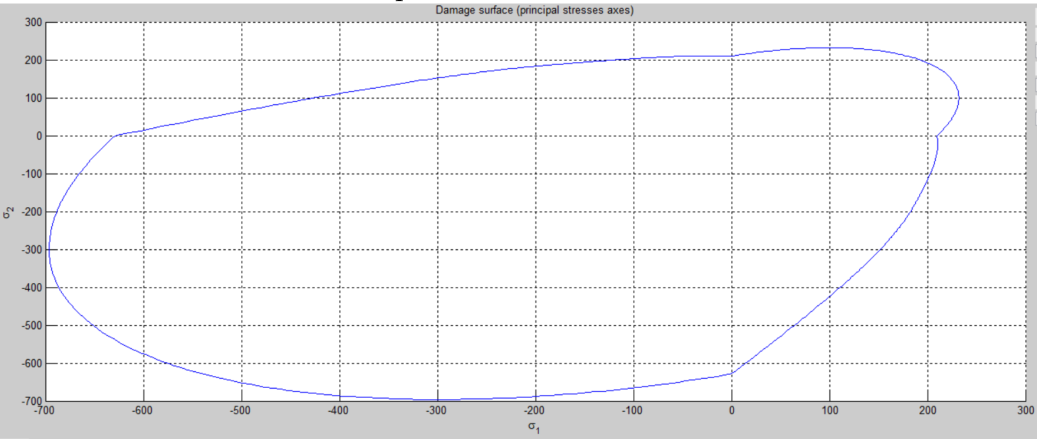

Figure 1: The "non-symmetric tension-compression damage" model

The "Tension-only" Damage Model

Unlike Symmetric Damage Model, in this kind of model the material fails only by tension. The code is available in Appendix A and the result is shown below. In this kind of model the elastic limit of compression cannot be reached. So in compression the elastic limit is at infinity. As seen from the figure, Damage is possible on by Tensile force.

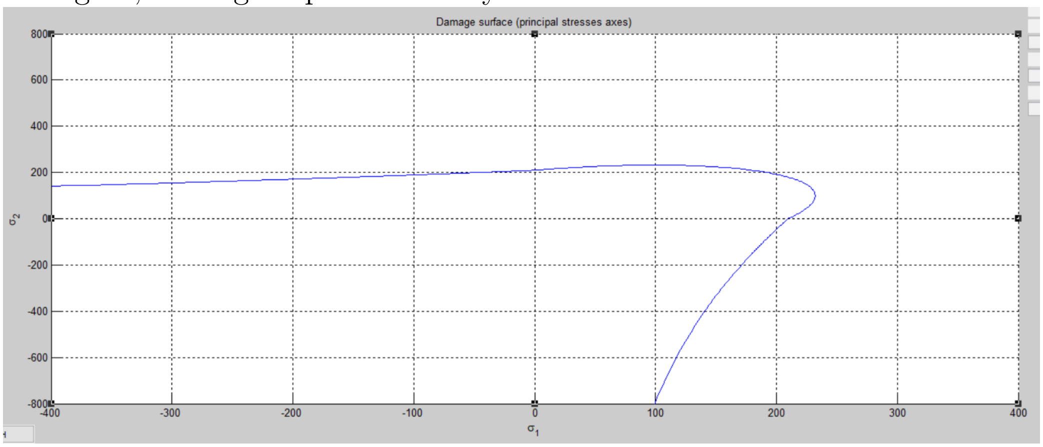

Figure 2: The "tension-only" damage model

Exponential Hardening/Softening Model

An Exponential Hardening/Softening model can never be negetive. It has been implemented using $q_\infty = 10^-6 r_o$. In case of Hardening $q_\infty > r_o$ and for Softening $q_\infty < r_o$. The code is available in Appendix B and the result is shown below.

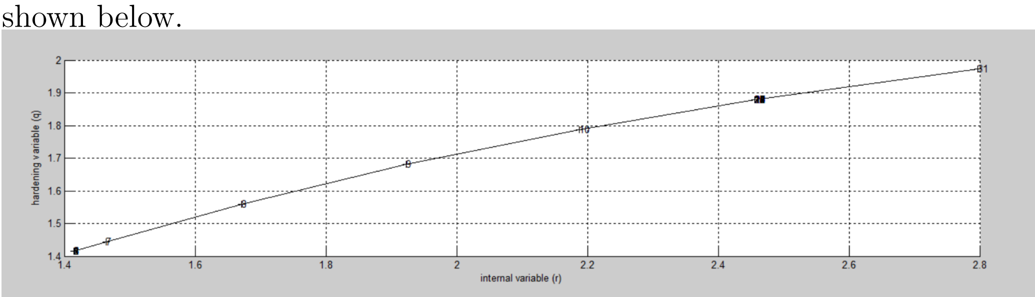

Figure 3: Exponential Hardening/Softening Model

Correctness of Implemented Algorithm

Case 1 - Non Symmetric Tension Compression Damage Model

The follwing values have been considered: ($\triangle\overline{\sigma_1} = 400$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = -1200$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = 300$, $\triangle\overline{\sigma_2} = 0$) The actual stress path becomes a straight line with a slope, till it reaches the origin. The evolution of the damage surface in the stress space has been shown in below.

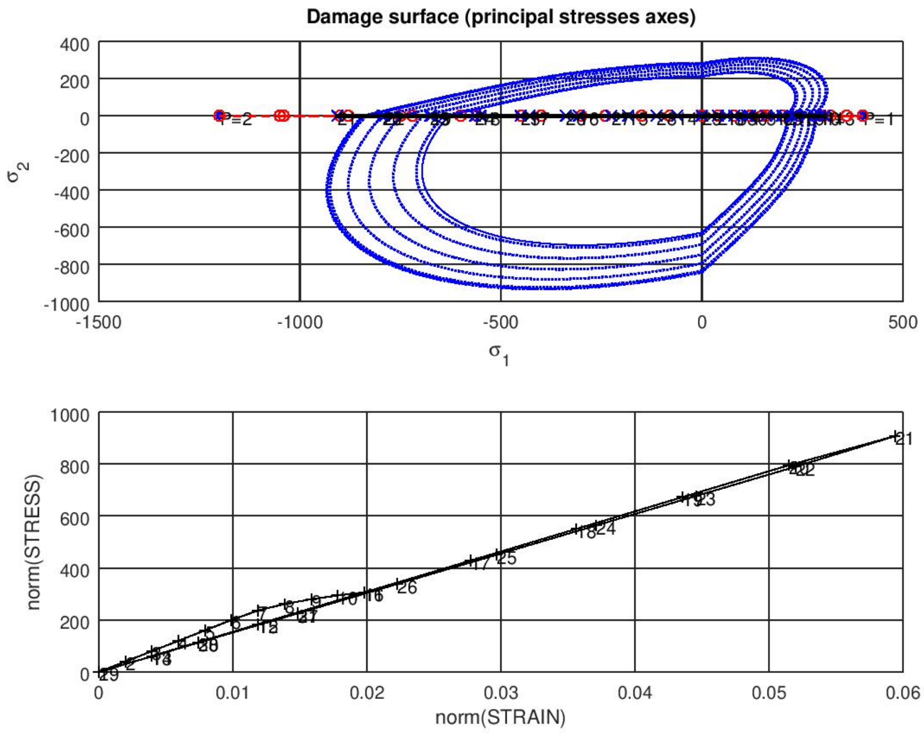

Figure 4: Case 1 - Non Symmetric

The follwing values have been considered: ($\triangle\overline{\sigma_1} = 400$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = -1200$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = 300$, $\triangle\overline{\sigma_2} = 0$)
The elastic limit in compression can't be reached. When the uniaxial elastic tensile loading occurs, the actual stress path again follows a straight line.

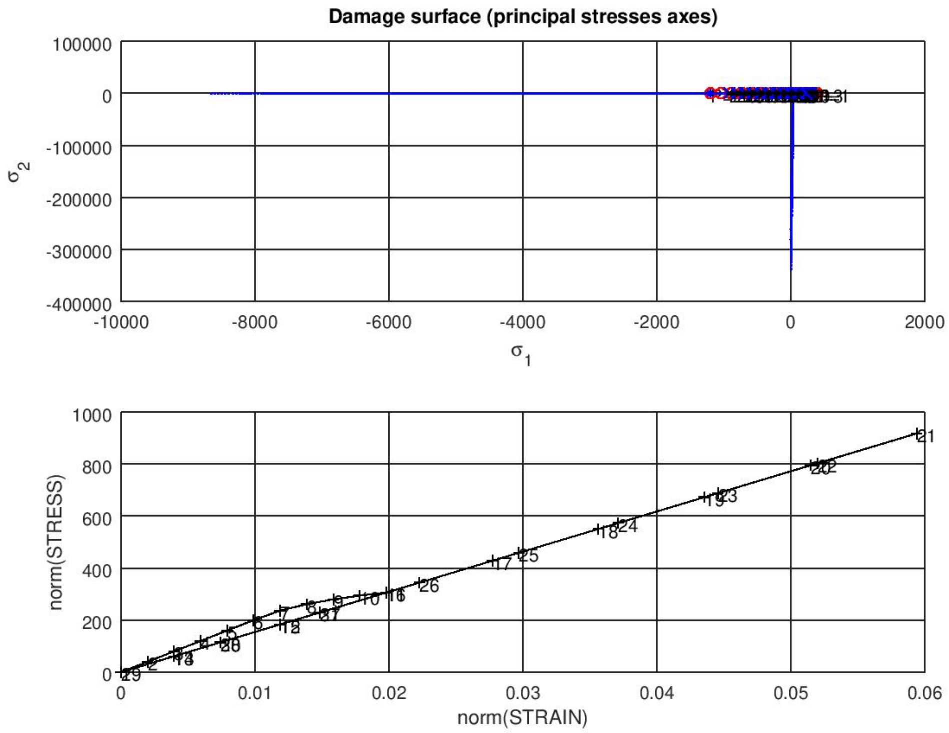

Figure 5: Case 1 - Tensile

The follwing values have been considered:($\triangle\overline{\sigma_1} = 200$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = -2800$, $\triangle\overline{\sigma_2} = -2800$), ($\triangle\overline{\sigma_1} = 600$, $\triangle\overline{\sigma_2} = 600$)

Upto elastic domain the uniaxial and elastic loading the effective and actual stresses are the same. When the stress path leaves the elastic limit of compression, loading occurs. After that biaxial compressive unloading/tensile loading takes place.

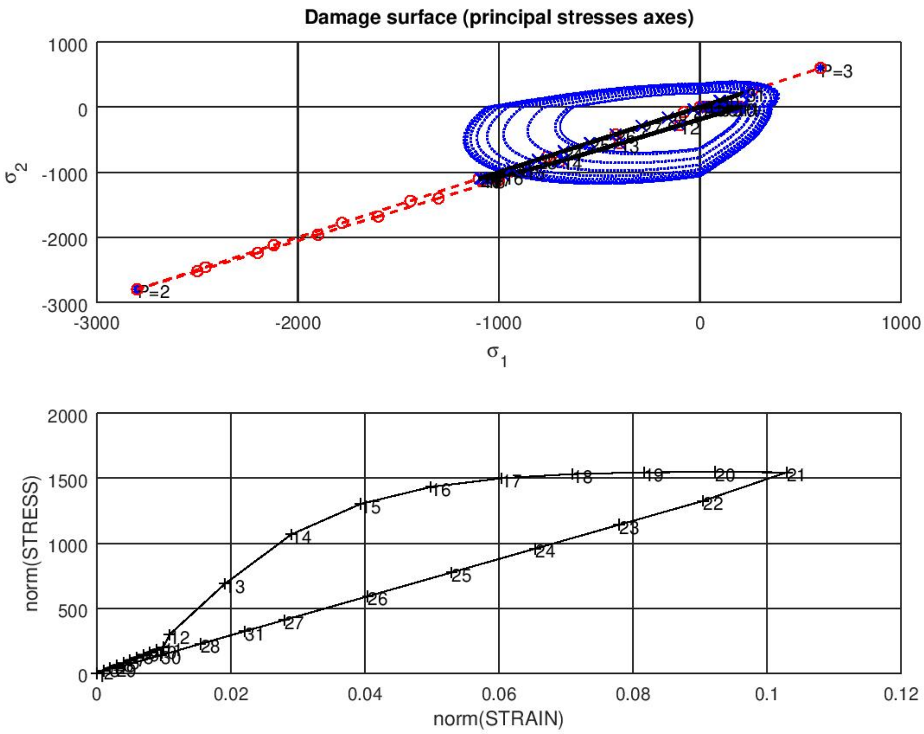

Figure 6: Case 2 - Non Symmetric

The follwing values have been considered: ($\triangle\overline{\sigma_1} = 200$, $\triangle\overline{\sigma_2} = 0$), ($\triangle\overline{\sigma_1} = -2800$, $\triangle\overline{\sigma_2} = -2800$), ($\triangle\overline{\sigma_1} = 600$, $\triangle\overline{\sigma_2} = 600$)

Upto elastic domain the uniaxial and elastic loading the effective and actual stresses are the same. When the stress path leaves the elastic limit of tension, loading occurs. After that biaxial compressive unloading/tensile loading takes place where biaxial elastic limit due to compression can never be reached.

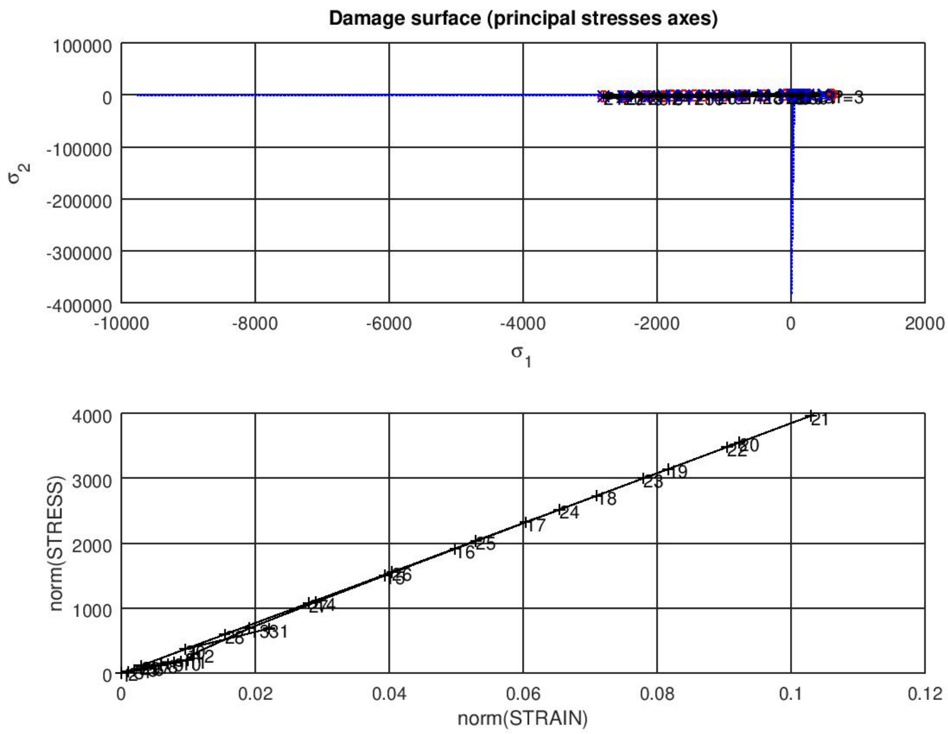

Figure 7: Case 2 - Tensile

The follwing values have been considered: $(\triangle\overline{\sigma_1} = 300,\ \triangle\overline{\sigma_2} = 300)$, $(\triangle\overline{\sigma_1} = -3000,\ \triangle\overline{\sigma_2} = -3000)$, $(\triangle\overline{\sigma_1} = 800,\ \triangle\overline{\sigma_2} = 800)$ Upto elastic domain the uniaxial and elastic loading the effective and actual stresses are the same. When the stress path leaves the elastic limit of compression, loading occurs. After that biaxial tensile unloading/compressive loading takes place. When the stress path leaves the elastic limit of compression, again loading occurs. After this the biaxial compressive unloading/tensile loading takes place.

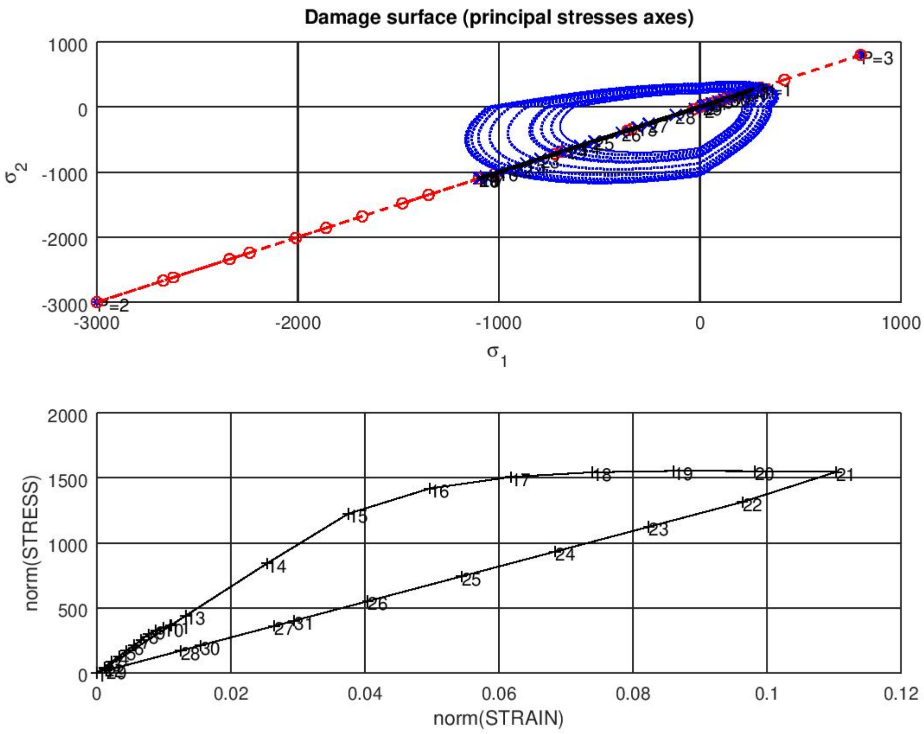

Figure 8: Case 3 - Non Symmetric

The follwing values have been considered: ($\triangle\overline{\sigma_1} = 300$, $\triangle\overline{\sigma_2} = 300$), ($\triangle\overline{\sigma_1} = -3000$, $\triangle\overline{\sigma_2} = -3000$), ($\triangle\overline{\sigma_1} = 800$, $\triangle\overline{\sigma_2} = 800$) Upto elastic domain the uniaxial and elastic loading the effective and actual stresses are the same. When the stress path leaves the elastic limit of tension, loading occurs. The biaxial elastic limit for compression can never be reached.

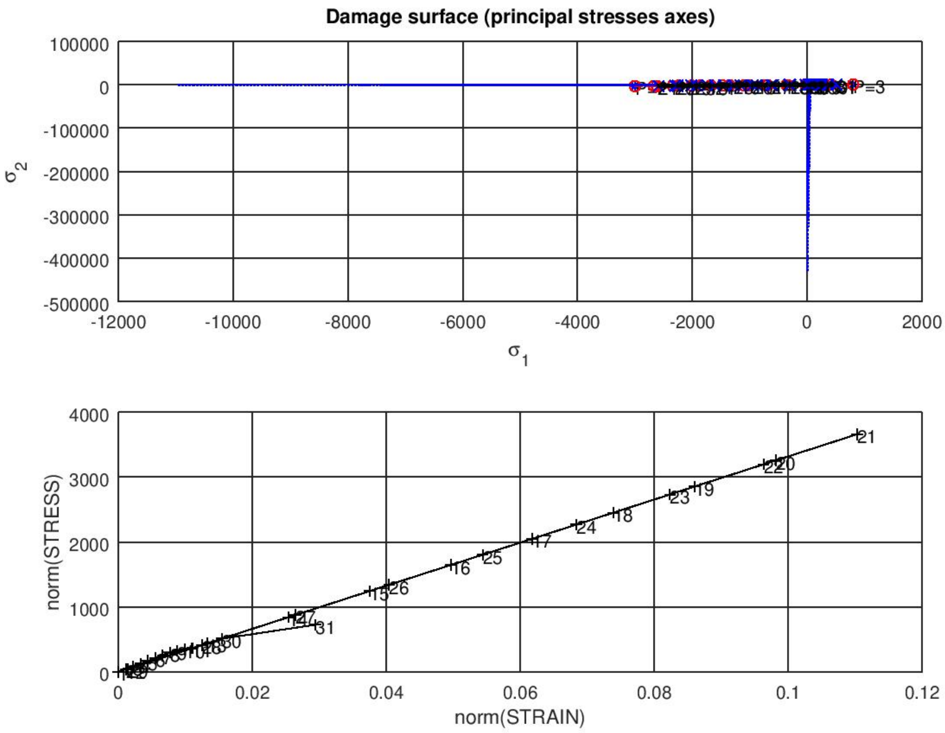

Figure 9: Case 3 - Tensile

Part II (Rate Dependent Models)

In this kind of models the rate effects are accomodated into the model. The stresses become dependent on both strains and rate of strains. The Code has been included in Appendix D.

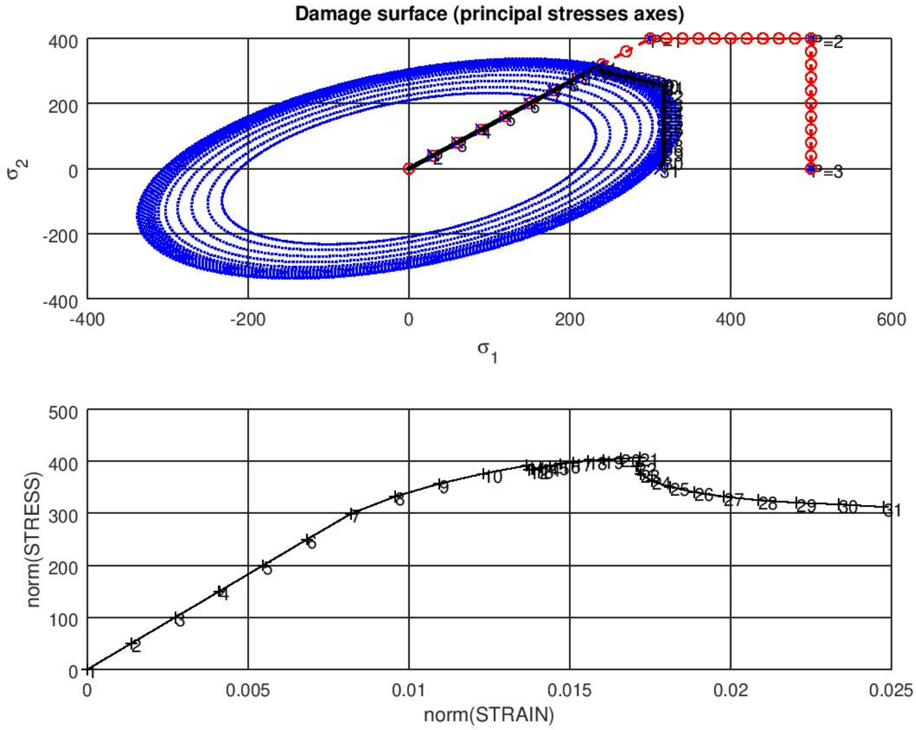

Figure 10: Visco-damage "symmetric tension-compression" model

Variation of Stress/Strain with viscosity η

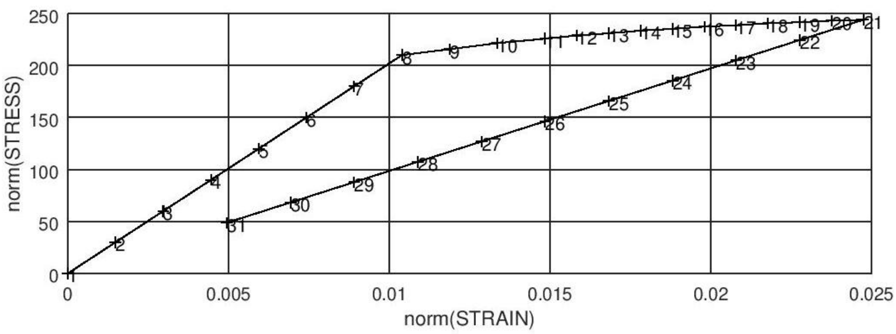

Figure 11: $\eta = 0$

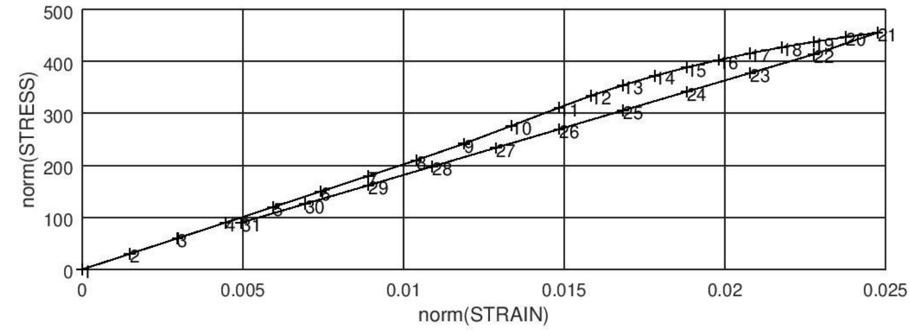

Figure 12: $\eta = 0.9$

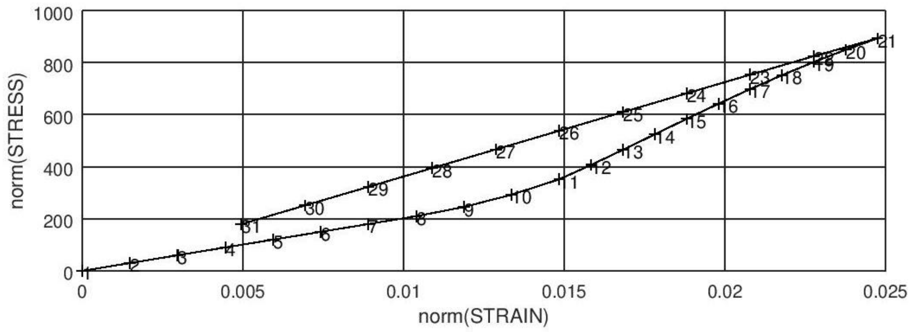

Figure 13: $\eta = 3$

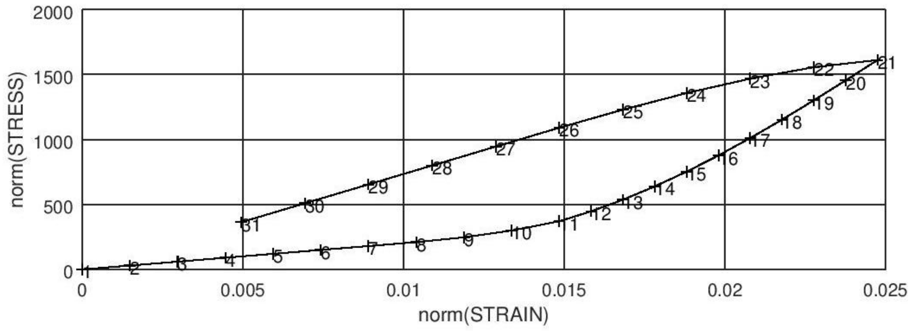

Figure 14: $\eta = 10$

From the above few figures it can be observed that viscosity effects are shown outside of the elastic domain. As viscosity increases so do the stress values increases.

Variation of Stress/Strain with Strain Rate $\dot{\xi}$

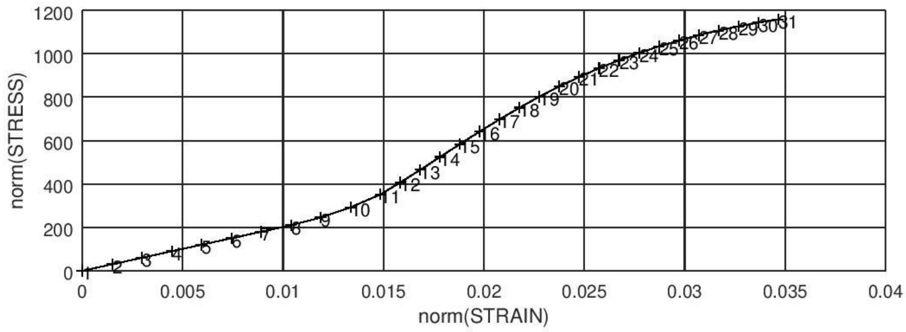

Figure 15: $\dot{\xi}_1$ for T=1

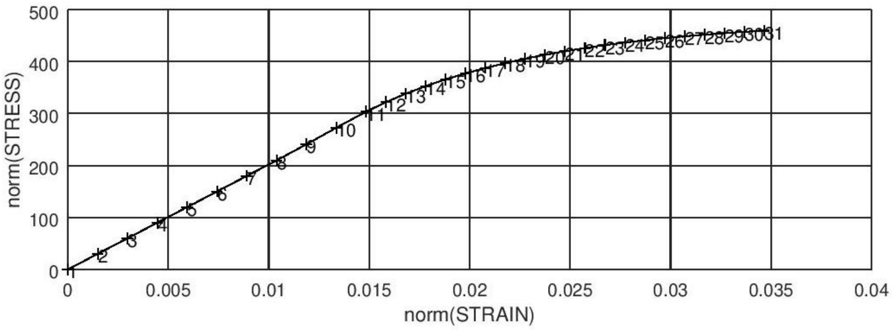

Figure 16: $\dot{\xi}_2$ for T=2

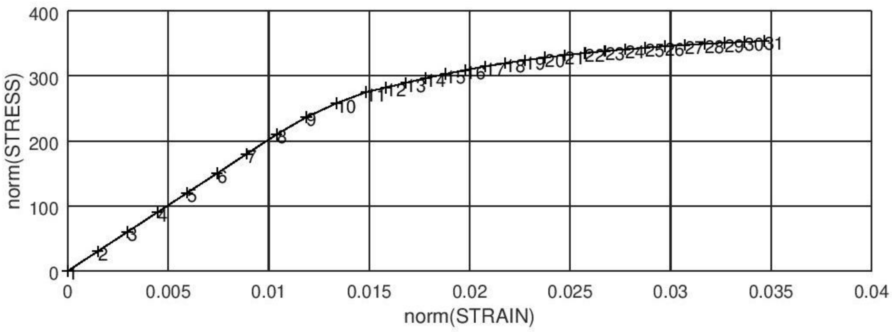

Figure 17: $\dot{\xi}_3$ for T=3

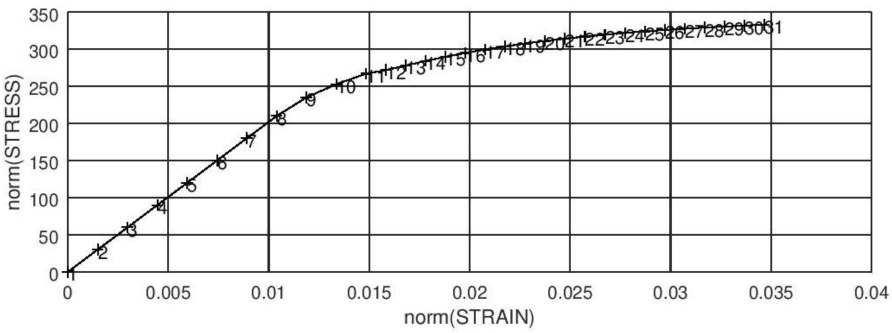

Figure 18: $\dot{\xi}_4$ for T=4

From the above few figures it can be observed that as strain rate increases, the stress at particular strain values also increases.

Variation of Stress/Strain with α

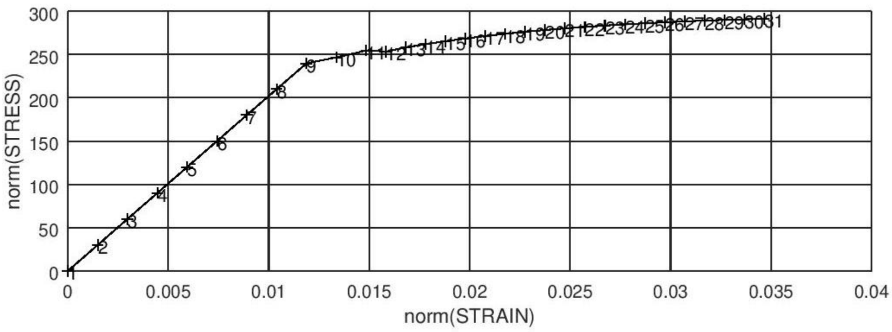

Figure 19: $\alpha_1 = 0$

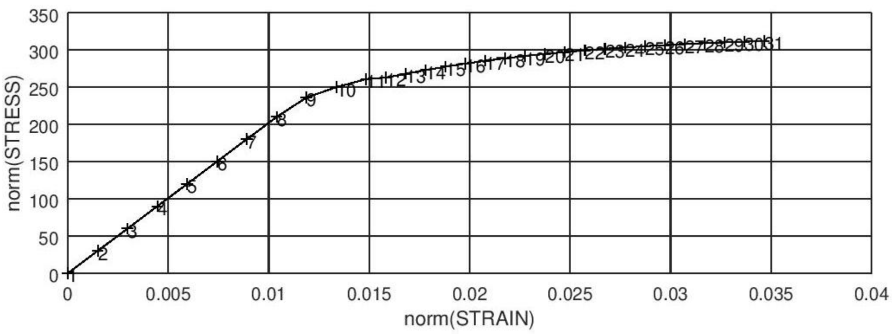

Figure 20: $\alpha_2 = 0.25$

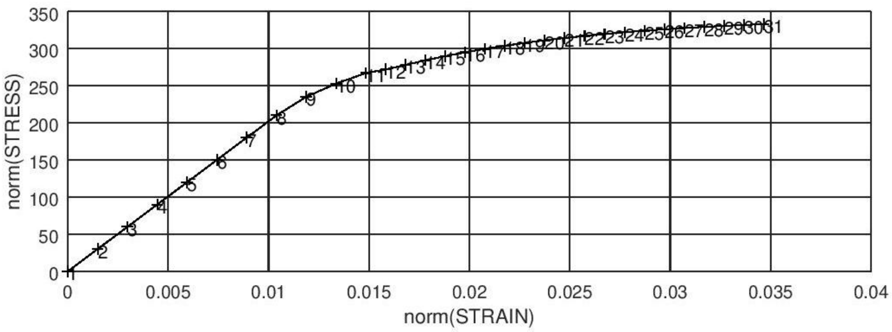

Figure 21: $\alpha_3 = 0.5$

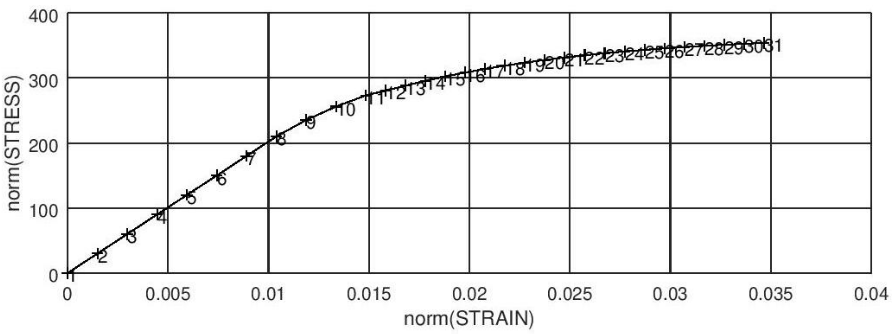

Figure 22: $\alpha_4 = 0.75$

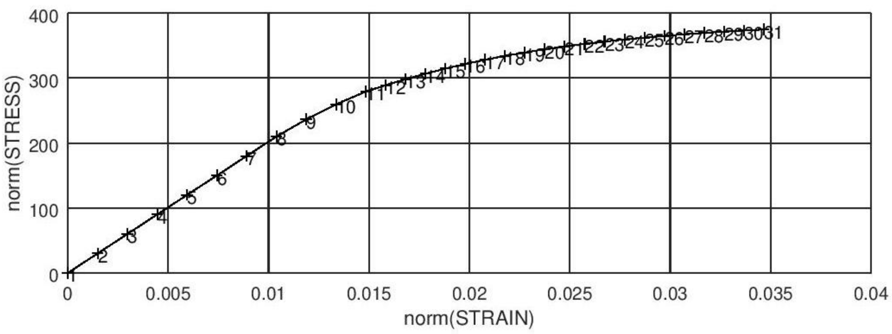

Figure 23: $\alpha_5 = 1.0$

For $\alpha >= 0$ and $\alpha < 0.5$ The methods are conditionally stable. For $\alpha = 0.5$ the method is unconditionally stable and is second order acurate. For all other values of $\alpha > 0.5$ the method is unconditionally stable and is first order acurate.

Variation of C_{11} component of C_{Alg} with α

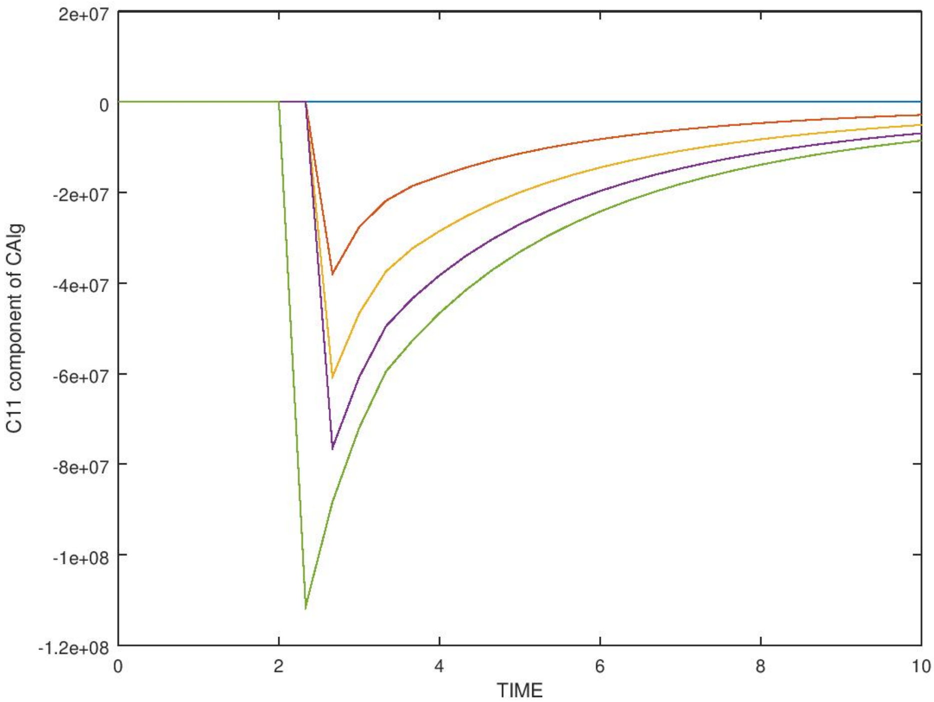

Figure 24: Effect of α on the C_{11} component of the tangent constitutive operators

It is observed that as within the elastic limit the value of C_{11} component of C_{Alg} remains the same. But outside of the same as the value of α increases the value of C_{11} decreases.

Variation of C_{11} component of C_{Tang} with α

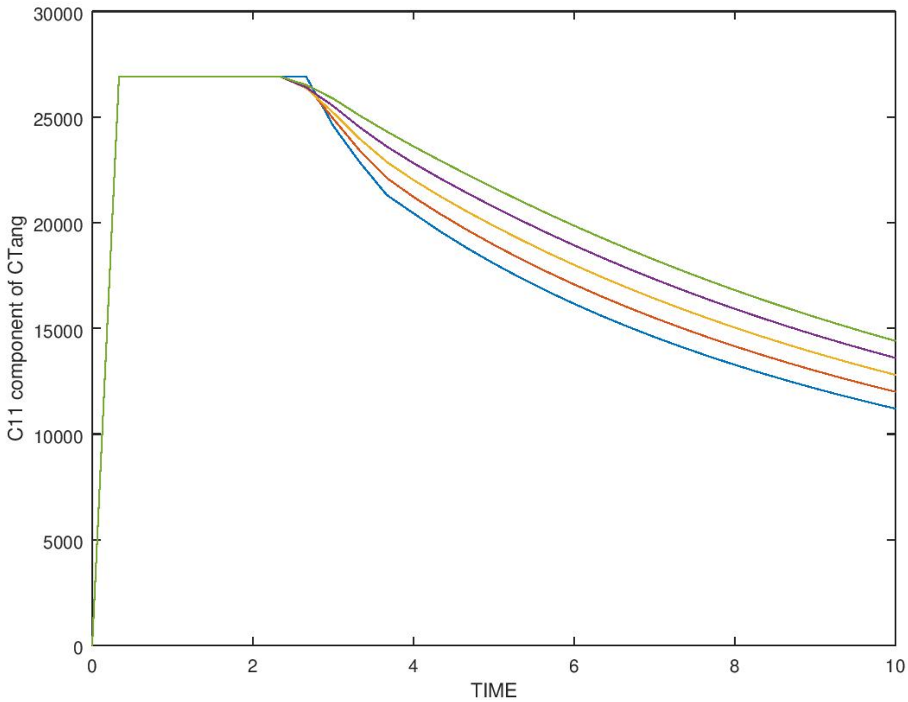

Figure 25: Effect of α on the C_{11} component of the algorithmic constitutive operators

It is observed that as within the elastic limit the value of C_{11} component of C_{Tang} remains the same. But outside of the same as the value of α increases the value of C_{11} increases.

Appendix

```
elseif (MDtype==2)  %* Only tension

sigmaPlus=macaulay(sigma);
rtrial= sqrt(sigmaPlus*eps_n1');

elseif (MDtype==3)  %*Non-symmetric
if (sum(arrayfun(@abs,sigma)))
tht=sum(macaulay(sigma))/sum(arrayfun(@abs,sigma));
else
tht=sum(macaulay(sigma));
end
rtrial=(tht+(1-tht)/n)*sqrt(eps_n1*ce*eps_n1');
```

Figure 26: Appendix A

```
if viscpr == 0
    if(rtrial > r_n)
        %*   Loading


        fload=1;
        delta_r=rtrial-r_n;
        r_n1= rtrial  ;
        if hard_type == 0
            %  Linear
            q_n1= q_n+ H*delta_r;
        else %exponential
            H_n1= A*(q_inf-r0)/r0*exp(A*(1-r_n/r0));
            q_n1= q_n+ H_n1*delta_r ;
        end
        if(q_n1<zero_q)
            q_n1=zero_q;
        end
    else
        %*     Elastic load/unload
        fload=0;
        r_n1= r_n  ;
        q_n1= q_n  ;
    end
```

Figure 27: Appendix B

```
    %added viscous model
else
    if (rtrial_n_alpha > r_n)
        %*   Loading
        fload=1;
        delta_r=rtrial_n_alpha-r_n;
        % r at the step n+1
        r_n1 = (eta - delta_t*(1-alpha))/(eta + alpha*delta_t)*r_n + (delta_t/(eta + alpha*delta_t))*rtrial_n_alpha;
        if hard_type == 0
            %  Linear
            H_n1 = H;
            q_n1= q_n+ H_n1*delta_r;
        else %Exponential
            H_n1= A*(q_inf-r0)/r0*exp(A*(1-r_n/r0));
            q_n1= q_n+ H_n1*delta_r ;
        end
        if(q_n1<zero_q)
            q_n1=zero_q;
        end
    else
        %*     Elastic load/unload
        fload=0;
        r_n1= r_n  ;
        q_n1= q_n  ;
    end
end
```

Figure 28: Appendix D