Coupled Problems: Homework

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Task 1

Solve a single heat transfer problem. The domain is [0,1]. Fix u=0 in both boundaries.

a)

Study the effect of changing the value to the thermal diffusion coefficient kappa.

The results obtained for different values of κ are depicted in figure 1. It can be concluded that when κ is decreased, the heat flux across the domain is faster, increasing the value obtained for u due to the source term s. However, for large values of κ there is more resistance and the value of u obtained is lower.

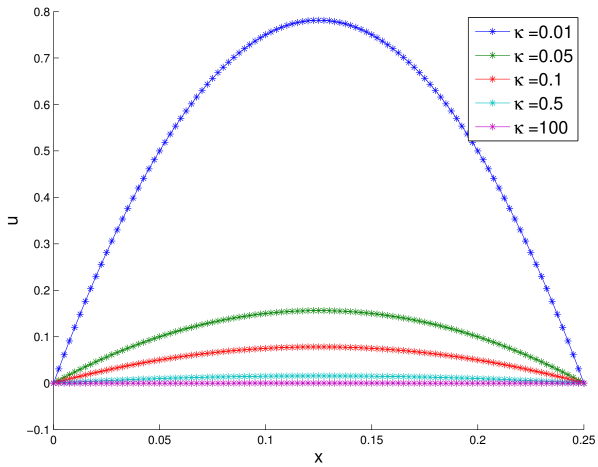


Figure 1: Results obtained for different values of κ, $s = 1$ and 100 elements

b)

Study the effect of changing the source term value.

The results obtained for different values of s are depicted in figure 2. The conclusions are opposed that those obtained for κ: when s is increased, the value obtained for u is increased since the source term is larger, and lower values of s produce smaller values of u.

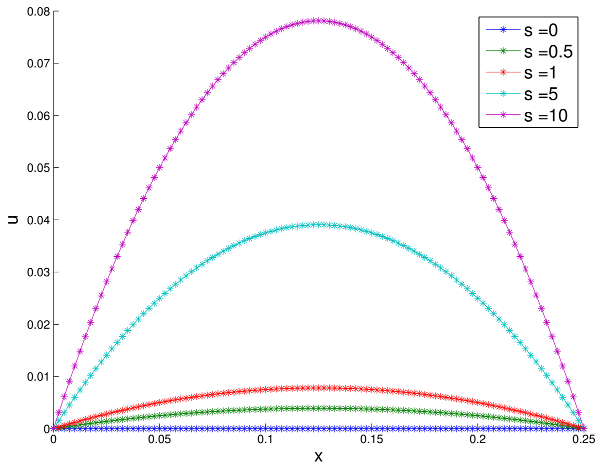


Figure 2: Results obtained for different values of source term s, $\kappa = 1$ and 100 elements

c)

Study the effect of changing the number of elements, evaluate the convergence rate of the error in the maximum heat value in the domain.

As can be seen in figure 3, the problem is so simple that is is converged even for the coarsest mesh used.

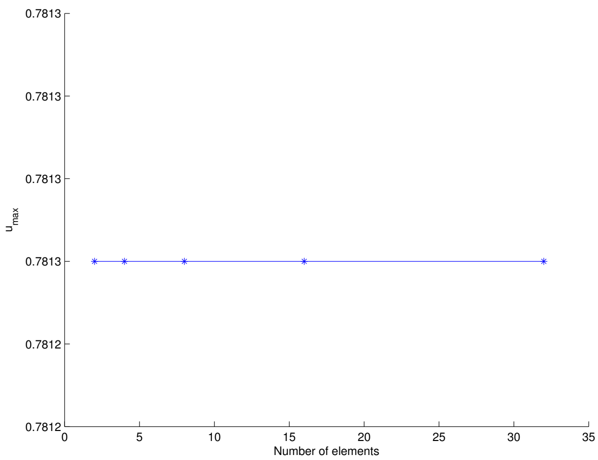


Figure 3: Maximum value of u for different number of elements and source term s=1 and $\kappa = 1$.

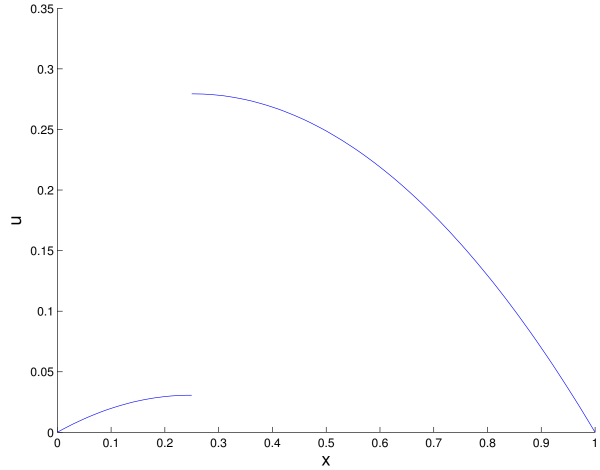


Figure 4: Results obtained solving the two subdomains separately.

Task 2

Solve two independent problems with kappa=1, source=1. The first problem subdomain is [0,0.25]. The second problem subdomain is [0.25,1]. Fix u in x=0 and x=1, leave it free in the interface between subdomains. Comment on the results.

If we solve two independent problems and do not enforce any transmission condition at the interface, we obtain two solutions which are discontinuous in terms of u (figure 4) and fluxes. This solution would not be an acceptable solution for a problem solved using a Domain Decomposition approach.

Task 3

Solve the previous problem in a Monolithic way.

a)

Study the "HP_SolveMonolithic.m" and relate it to what was explained in theory. Comment on the results.

In the file "HP_SolveMonolithic.m" we are solving the problem as a single system of equations:

$$\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Where K_1 and K_2 are the matrices that are obtained at every subdomain. The degree of freedoms at the interface has contributions from both subdomains since it has influence over them.

The results obtained when solving the problem using 25 elements for subdomain 1 and 75 elements for subdomain 2 are depicted in figure 5, and are very similar to those obtained when solving for the whole domain.

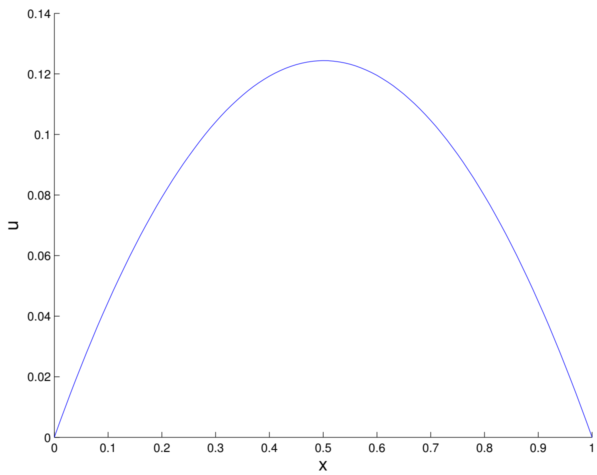


Figure 5: Results obtained using a monolithic approach.

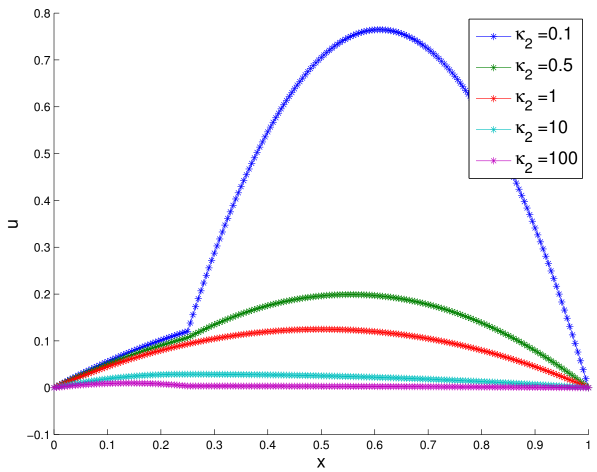


Figure 6: Results obtained $\kappa = 1$ at the left subdomain and different values of κ at subdomain 2.

b)

Modify the kappa parameter of one of the subdomains. Comment on the results.

The results obtained for different values of κ at subdomain 2 are depicted in figure 6. As we can see, the continuity of u is satisfied at the interface, but having different values of κ changes the heat transfer across the domain and the slope of u for a given flux.

Task 4

Solve the previous problem (kappa=1 in both subdomains) in an iterative manner (Dirichlet Neumann). Apply Neumann boundary conditions at the interface in the first subdomain (left) and Dirichlet at the interface in the second subdomain.

a)

Evaluate the convergence of the iterative scheme (in terms of u at the interface).

An iterative scheme has been implemented using a Dirichlet-Neumann approach. The solution obtained for 100 elements (figure 7) is similar to the solution obtained solving the problem using a single subdomain. The evolution of the relative error computed as

$$E = \frac{\|u_{left} - u_{right}\|}{\|u_{left}\|}$$

is depicted for both fluxes and u in figure 7. As can be seen, the results obtained for the continuity of u are very good and are satisfied almost from the first iteration. However, the error obtained for the flux at the interface are larger and take more time to converge. Thus, the Neumann step is the one that has more influence over convergence. It can be seen that the finner is the mesh, the faster is the convergence for the fluxes: For a finner meshes we have a better approximation of the derivatives.

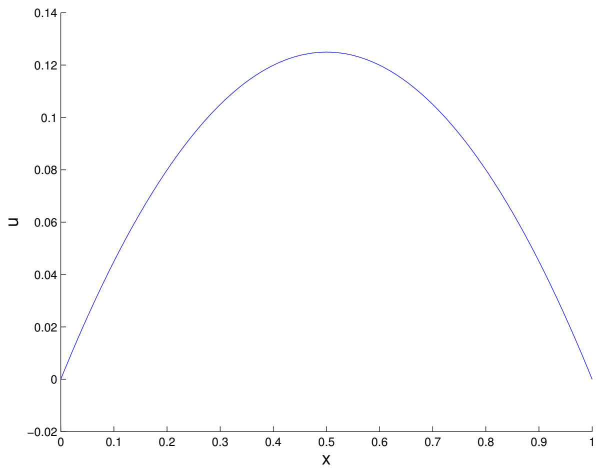
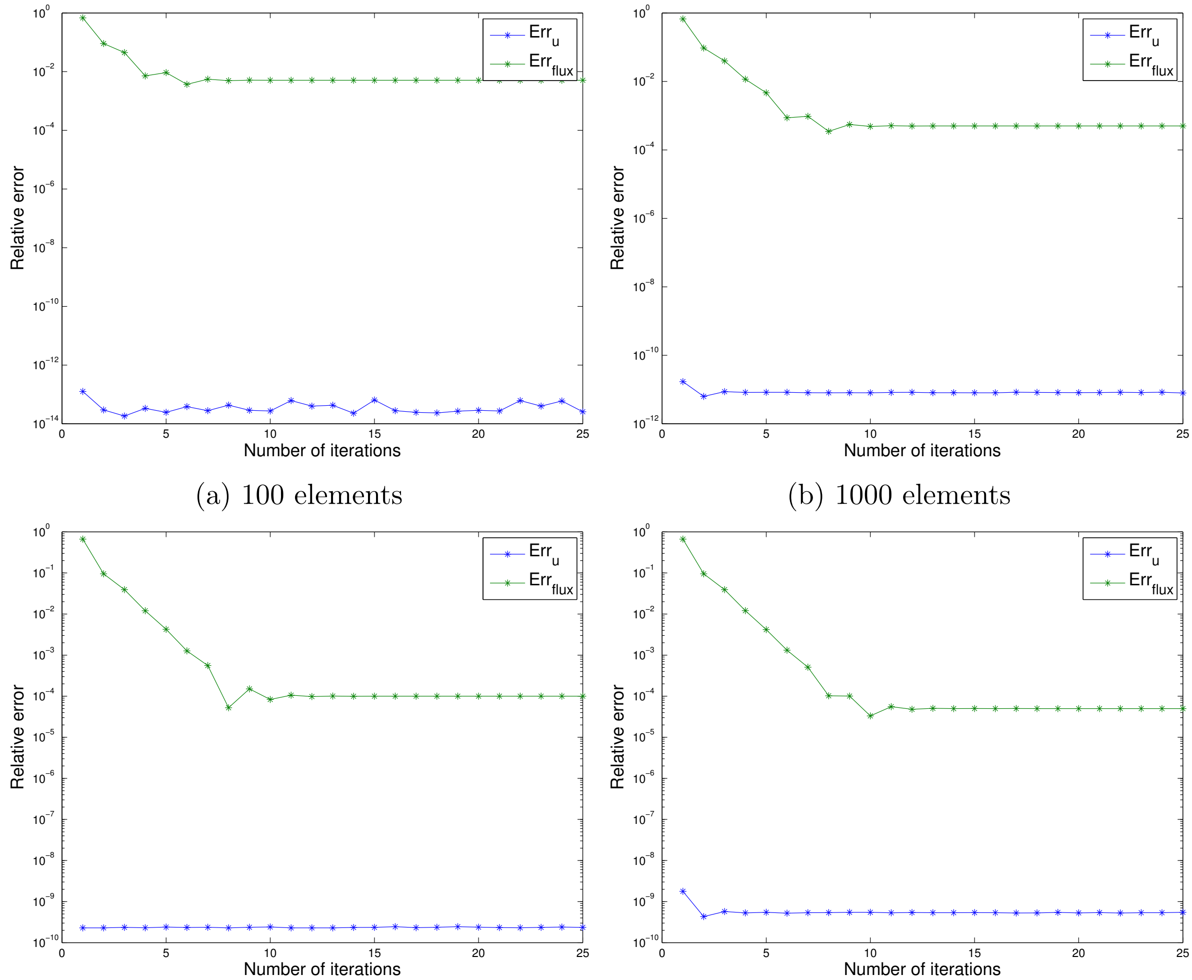


Figure 7: Solution for the Dirichlet-Neumann scheme and 100 elements.



(a) 100 elements (b) 1000 elements (c) 5000 elements (d) 10000 elements

Figure 8: Convergence for the Dirichlet-Neumann scheme and different number of elements

b)

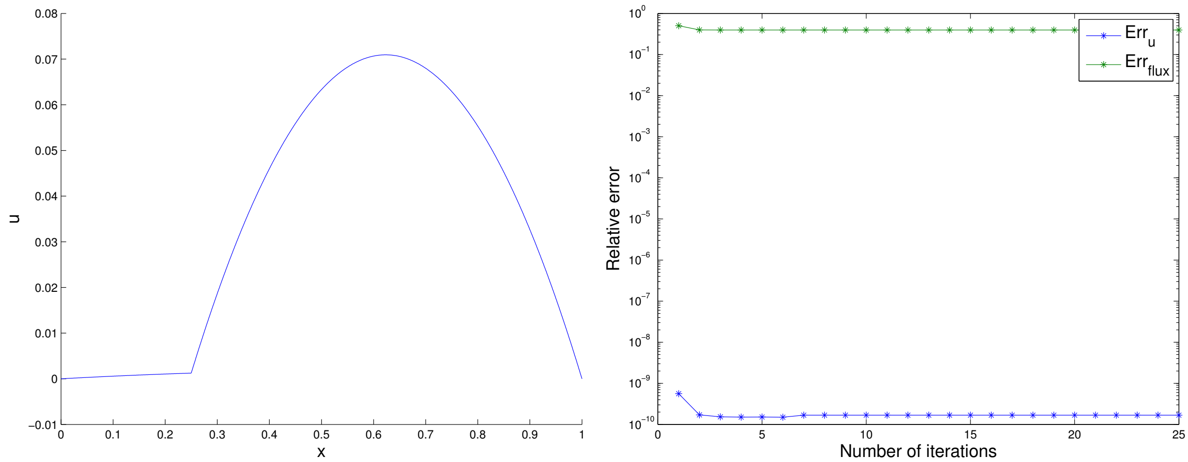
Increase the value for kappa at subdomain 1 (x100). Comment on the convergence rate.

If we increase the value for κ at subdomain 1, we have that the continuities of fluxes is ensure when the derivative of u at subdomain 2 is much larger than the derivative at subdomain 1 (figure 9 a). This makes harder to achieve convergence for the fluxes. However, the results obtained are stable.

c)

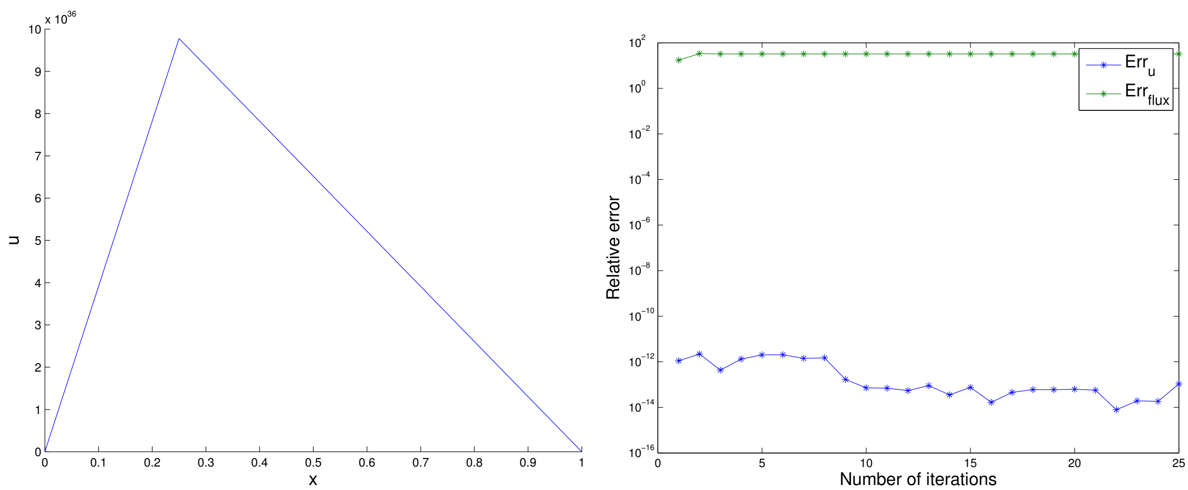
Diminish the value for kappa at subdomain 1 (/100). Comment on the convergence rate.

If we set $\kappa_1 = 0.01 = \frac{\kappa_2}{100}$ at subdomain 1, we have that the continuities of fluxes is achieved when the derivative of u at subdomain 1 is much larger than the derivative at subdomain 1 (figure 9 a). However



(a) Solution (b) Convergence

Figure 9: Results for the Dirichlet-Neumann scheme, 1000 elements and $\kappa = 100$ at subdomain 1



(a) Solution (b) Convergence

Figure 10: Results for the Dirichlet-Neumann scheme, 1000 elements and $\kappa = 0.01$ at subdomain 1

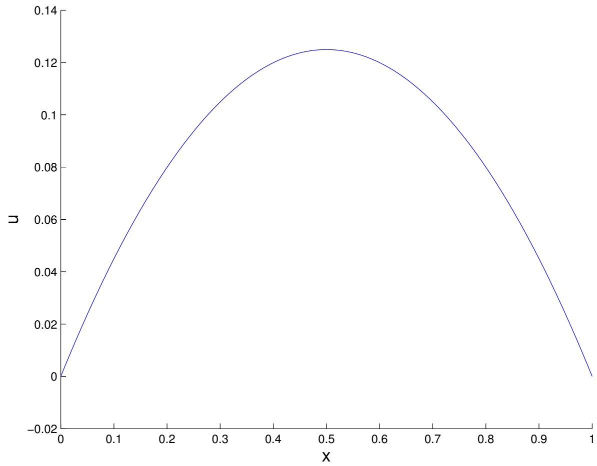


Figure 11: Solution for the relaxation scheme and 1000 elements.

d)

Motivate the previous results in terms of the stability of the coupling scheme.

The coupling scheme is stable when κ_1 and κ_1 have similar values because we have homogeneus problems. However, when $\kappa_1 << \kappa_2$, the continuity of fluxes means that we are passing a huge derivative of u to subdomain 1, which can reduce the coercivity of the system and substract stability.

Task 5

Implement a relaxation shceme.

a)

Relaxation scheme in terms of a fixed relaxation parameter w.

A relaxation scheme has been implemented using a relaxation parameter w which affects the Dirichlet condition imposed at subdomain 2:

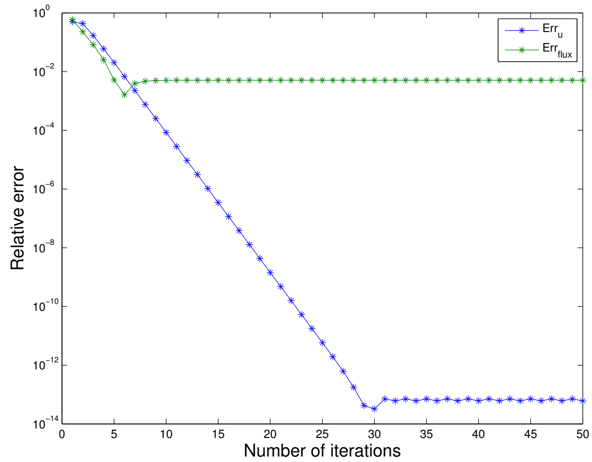
$$u_{right}^{k} = w \cdot u_{right}^{k} - (1 - w) \cdot u_{right}^{k-1}$$

This why, any unstability obtained can be relaxed and reduced. The results obtained when using this scheme are depicted in figures 11 and 12. We see a slower convergence rate, but the error obtained for both the fluxes and u is lower.

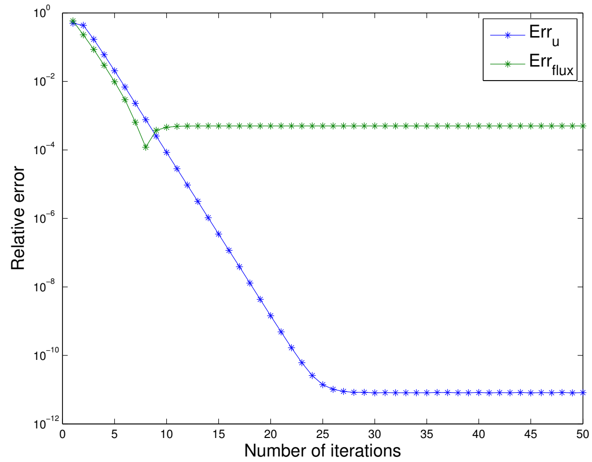
b)

Aitken relaxation scheme.

The Aitken relaxation scheme has been implemented. This scheme is similar to last scheme, but the relaxation term α is not constant and depends on the error obtained in the previous iteration. When using this scheme (figures 13 and 14), we obtain a much better convergence that for constant relaxation parameter. Moreover, the scheme is more stable.



(a) 100 elements



(b) 1000 elements

Figure 12: Convergence for the relaxation scheme w and different number of elements

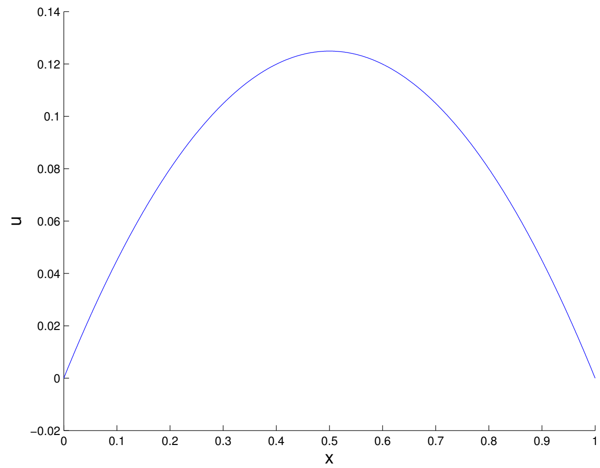
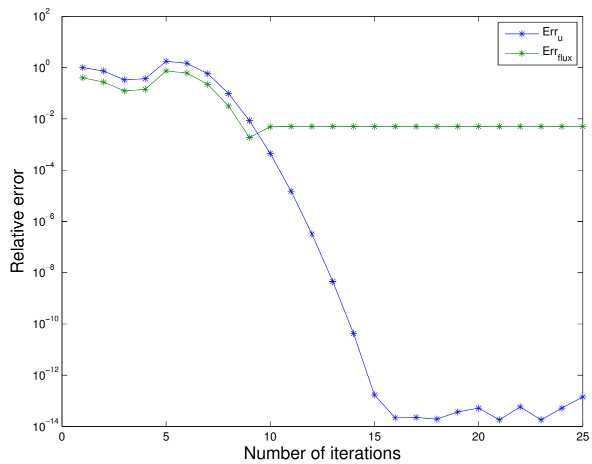
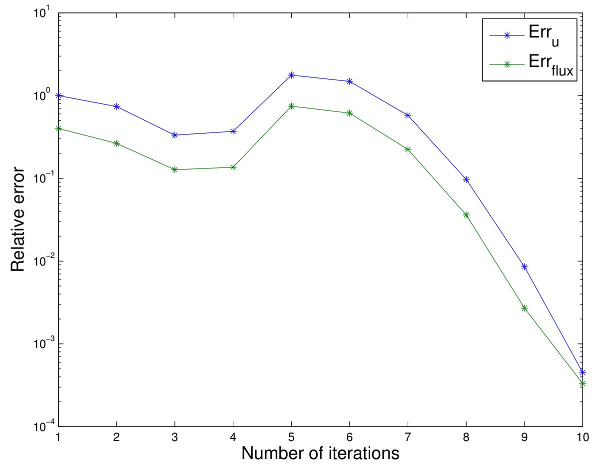


Figure 13: Solution for the Aitken relaxation scheme and 1000 elements.



(a) 100 elements



(b) 1000 elements

Figure 14: Convergence for the Aitken relaxation scheme and different number of elements