Numerical Homework

Coupled Problems
Assignment - 2

by

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Task 1: Solve a single heat transfer problem

1. Study the effect of changing the value to the thermal diffusion coefficient kappa.

   Different kappa values were used in order to show the behaviour of the temperature solution. The solution computed is shown in figure below.

   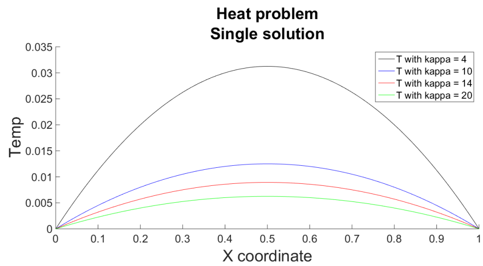

   Figure 1: Temperature result for different kappa values

   It can be observed that while kappa increases, the maximum temperature decreases. Therefore, it can be said, that materials with low diffusivity coefficient are not able to expulse heat outside of the domain in a fast way, therefore, the temperature start to rise until get equilibrium. As contrary, materials with high diffusivity coefficient are able to expulse heat outside of the domain in a fast way, as consecuences, the temperature diminishes until get a stable point.

2. Study the effect of changing the source term value.

   Different source values were used in order to show the behaviour of the solution. It can be obsered, in fugure 2, different solutions for each source value.

   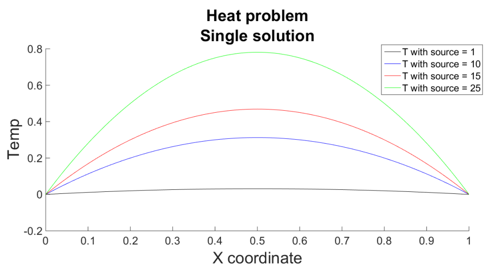

   Figure 2: Temperature result for different source term values

   While source term value (volume heat) increases, the temperature responce increases as well.

3. Study the effect of changing the number of elements, evaluate the convergence rate of the error in the maximum heat value in the domain.

   In this part of the homework, different element numbers were used. Figure 3 (Left) shows diffenret solutions for different element numbers. On the other hand, in figure 3 (Right) can be seen how the absolute error of the solution increases when mesh size increases as well. The solution obtained with 100 element was taken as exact solution, and mesh size was computed as the length of the domain divided by element numbers. The absolute error was calculated taking into account the maximum heat value of the domain.

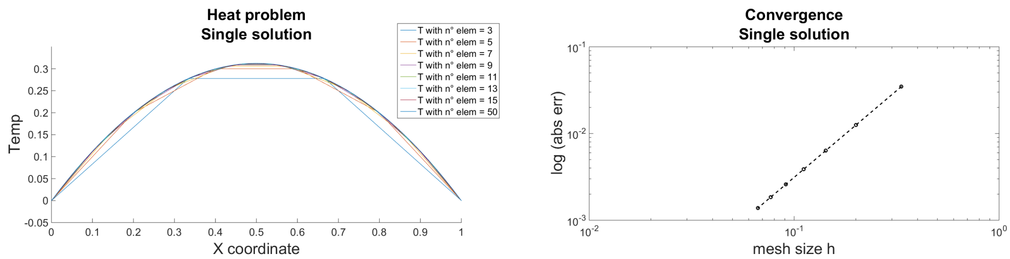

Figure 3: Result for different element numbers: Left: Temperature solution for different element numbers Right: Convergence of the solution vs element numbers.

Since linear elements was used , convergence was order 1. Second, while the number of elements increases (mesh size decreases) the absolute error decreases.

Task 2: Solve two independent heat transfer problems

1. It was used the same kappa value for the two subdomains. The first problem subdomain is [0, 0.25]. The second problem subdomain is [0.25,1].

   Nex figure shows the result obtaned.

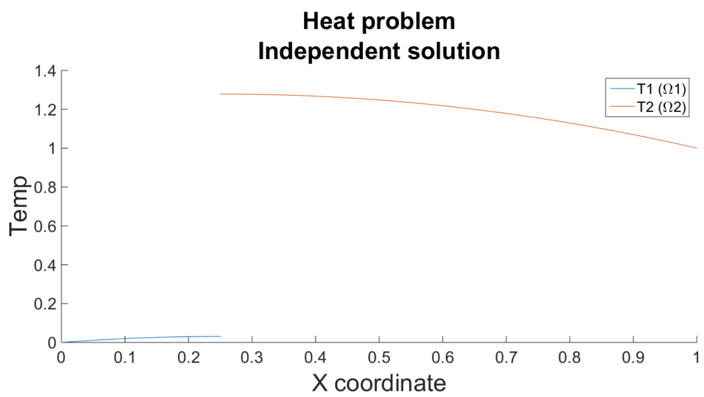

Figure 4: Different subdomain results

Since the interface of subdomains was left free, there was no tramsmission condition, therefore, each subdomain was solved as a dirichlet problem without any transmission between them. It can be observed that the temperature was discontinuous on the interface between subdomains.

Task 3: Solve the previous problem in a Monolithic way

1. It was used the same kappa value for the two subdomains. The first problem subdomain is [0, 0.25]. The second problem subdomain is [0.25,1].

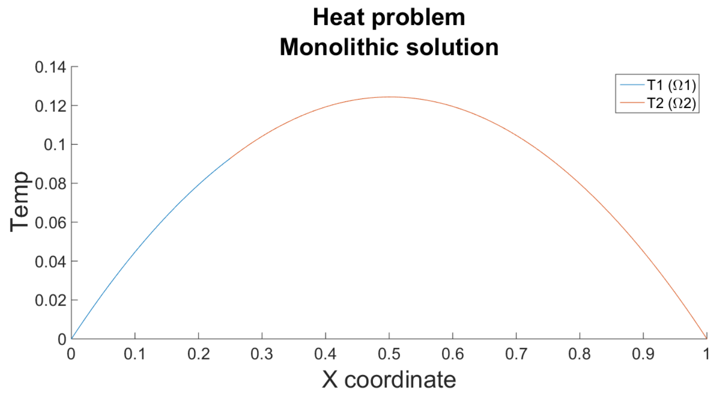

Figure 5: Monolithic solution

It can be osberved that the transmition conditions were satisfied, not only continuity of temperature but also, continuity of fluxes in the interface between subdomains.

2. Modify the kappa parameter of one of the subdomains.

   It was modified kappa of subdomain 1. The kappa value was equal to 4. In the figure below can be seen the result.

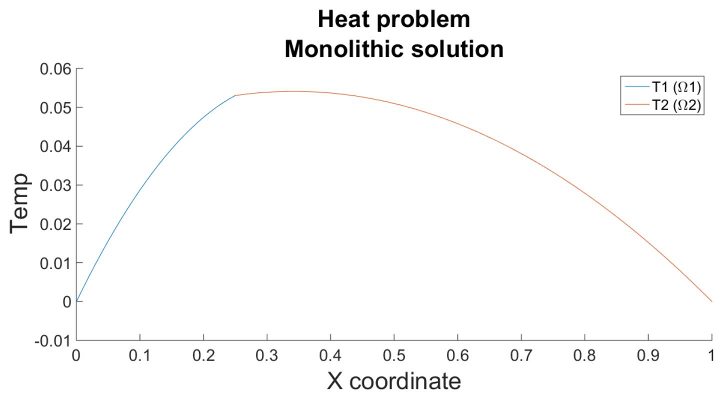

Figure 6: Monolithic solution: Different kappa value for each subdomain

   As expected, once kappa value changed, the fluxes becomes discontinuous. It can be observed that at x = 0.25, the derivative of the solution has two values, depending in which domain is computed.

$$k_1 \frac{\partial T}{\partial x}\Big|_{x=0.25} \neq k_2 \frac{\partial T}{\partial x}\Big|_{x=0.25} \tag{1}$$

Task 4: Solve the previous problem (kappa = 1 in both subdomains) in an iterative manner (Dirichlet-Neumann). Apply Neumann boundary conditions at the interface in the first (left) subdomain, and Dirichlet boundary conditions at the interface in the second subdomain.

1. Evaluate the convergence of the iterative scheme (in terms of T at the interface)

   Figure 7 (Left) shows the solution obtaind by iteration-by-subdomain. The left subdomain $\Omega 1$ was solved as a Neumann problem, while, the rigth subdomain $\Omega 2$ was solved as a Dirichlet problem. On the other hand, it can be seen, in figure 7 (Right), the convergence of the methods, error vs iteration numbers.

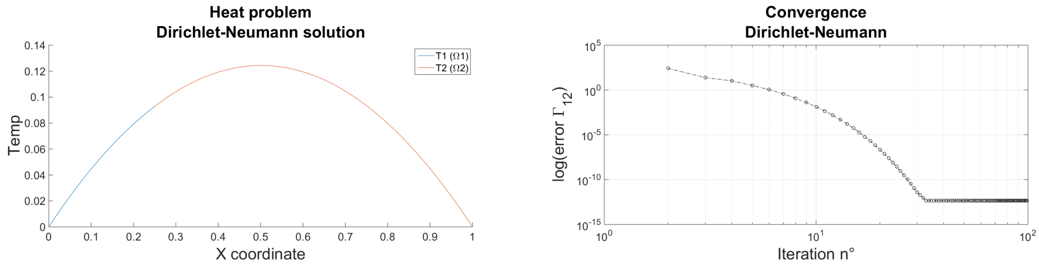

Figure 7: Iteration by subdomain. Left: Solution, Right: Error at the interface vs iteration number

   It can be observed, how the error decreases while iteration number increases. The error is decreasing monotonic and at iteration 15th the solution is reached.

2. Increase the value of kappa by 100 at subdomain $\Omega 1$.

   The figure below shows how the solution changes as consequence of using different kappa values on each subdomain, also, it can be seen the convergence of the method.

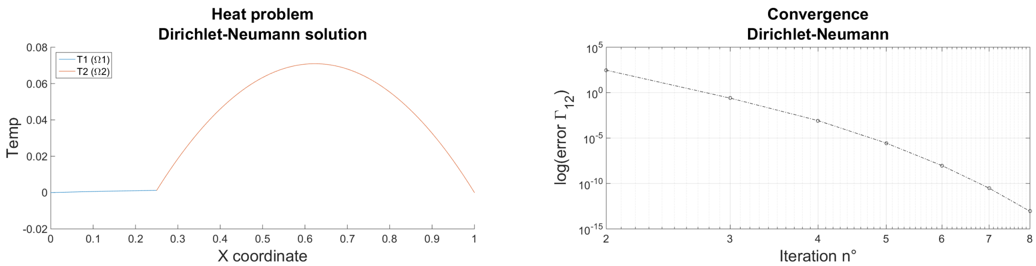

Figure 8: Iteration by subdomain. Left: Solution, Right: Error at the interface vs iteration number

   From the iteravite point of view, the solution is reached faster, it takes only 8 iteration. But from the numerical point of view, the method has obtained a wrong solution. It can be observed, in the next equation evaluated on the boundary, if $k_1 > k_2$ the solution is reached in a few iteration, but a wrong solution is obtained.

$$\frac{\partial T^{(k+1)}}{\partial x}\Big|_{x=0.25} = \frac{k_2}{k_1}\frac{\partial T^{(k)}}{\partial x}\Big|_{x=0.25} \tag{2}$$

3. Diminish the value of kappa by 0.01 at subdomain $\Omega 1$.

   As a consequence of diminishing kappa, it can be noted that the solution has blown up. The method never reached the convergency, therefore, the solution plot has not been shown. As interesting remark, once the method has reached 10 iteration, error has became larger and remained constant along iteration numbers, therefore, the method never converge.

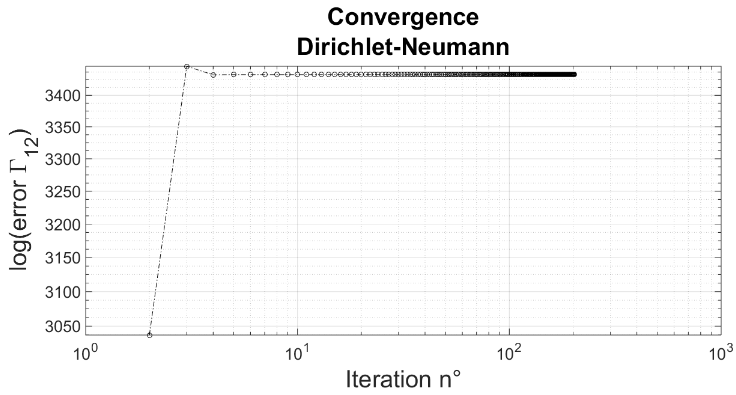

Figure 9: Error at the interface vs iteration number

4. Stability of the coupling scheme.

   As final remark, the scheme presented some inestabilities for small kappa values (0.01), this produced that the error on the interface increased through iteratives number,and the method never reached the converge. On the other hand, for high kappa values, the method showed some stability, but the a wrong solution was obtained.

Task 5: Implement a relaxation scheme.

1. Relaxation scheme in terms of a fixed relaxation parameter w.

   Figure 10 (Top Left) shows the solution obtaind by relaxation scheme and the last figures show the error at interfaces between subdomain vs iteration number, using different relaxation value.

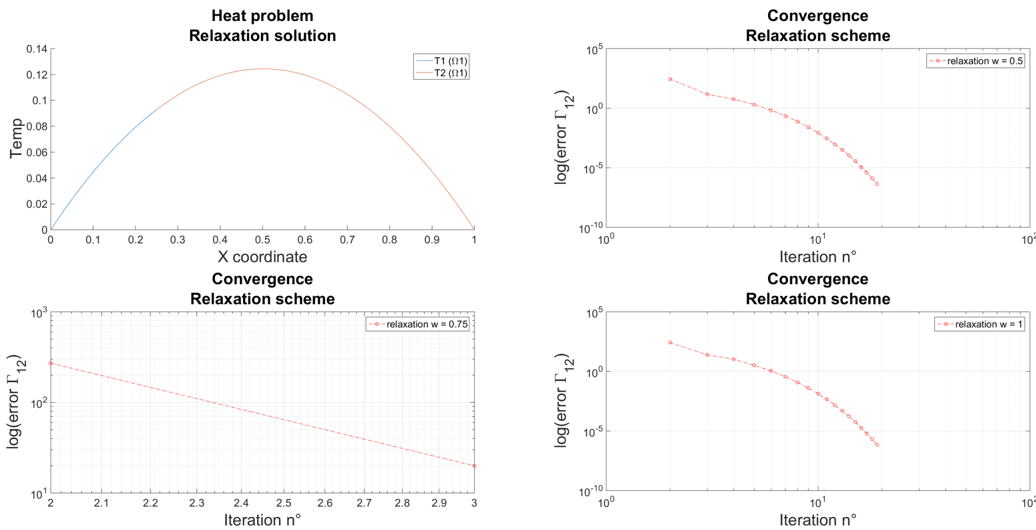

Figure 10: Relaxation scheme. Top Left: Solution, Top Right: Error at ω=0.5, Bottom Left:Error at ω=0.75 Optimal value, Error at ω=1

As expected, the optimal value of relaxation coefficient was equal to 0.75. For ω value equal to 0.75, the scheme only needed three iteration until get the solution. Otherwise, for different ω values, the system obtained the solution but using more than three iterations. As a particular case, for ω=1, iteration-by-subdomain Dirichlet-Neumann is recovered.

2. Aitken relaxation scheme.

   In this task, the solution has been computed using Aitken relaxation scheme. It can be observed, in figure 11, the convergence plot of the method and the its solution.

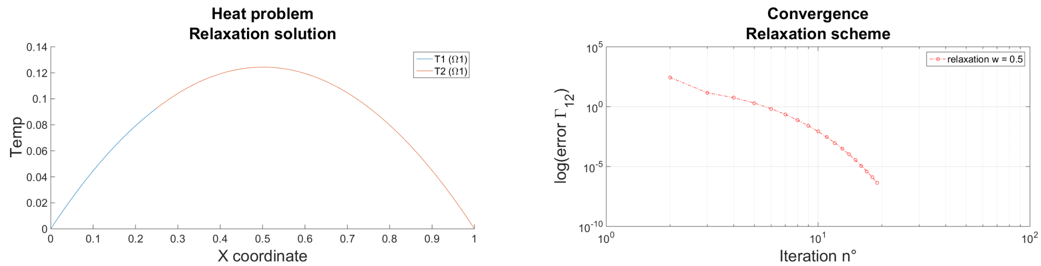

Figure 11: Aitken relaxation scheme. Left: Solution, Right: Error at the interface vs iteration number for different ω values

This method has some particularities. First, it uses information of the previous iterations. Once the second iteration is computed, Aitken obtains a new value for omega. For the next iteration Aitken compute, again, a new parametere, and so on. Finally, the omega value always is optimal in order to use as less amount of iteration as possible.