

Computational Mechanics Tools: Assignment 2

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2nd December 2015

1. The problem is solved using the MATLAB PDE toolbox and following the steps of the example made in class. Once the geometry is drawn, the heat PDE is defined (Parabolic PDE) and are imposed the boundary conditions given by the statement; the problem is defined. As the class example was time-independent, in this case we have to add the initial conditions using `Solve > Solve PDE`. The solution of the problem is the following:

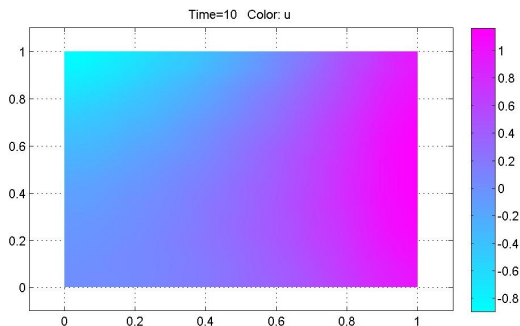


Figure 1: Temperature distribution at $t_{end} = 10$ with mesh size $h = 0,1$.

Now it is time to refine the mesh, and is done changing the maximum edge size with `Mesh > Parameters`. The problem is solved with mesh sizes 0,1, 0,05, 0,025 and 0,0125. With every solution the mesh is exported to MATLAB with `Mesh > Export Mesh...` and the temperatures with `Solve > Export Solution...`. Now using the given `errorExample1.m` file, the error is computed compared to the analytic solution, given by the statement. Then, the error is plotted with different mesh sizes using the `convergenceExample1.m` file:

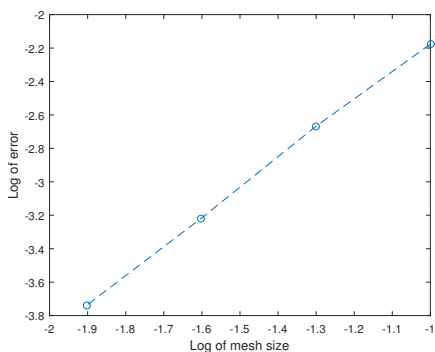


Figure 2: Log-log plot of error vs mesh size.

It is observed that the logarithm of the error follows a straight that depends on mesh size, so the error obtained is bounded, as the convergence theory explains. In this case, using the linear regression of the data on Figure 2, is obtained that the error, for little mesh sizes, behaves as $\|e\|_{\infty} \approx C h^{1,7402}$ that is the boundary of the error.

2. In this case, simulation time is changed with `Solve > Parameters...` and is used a time step of 1 to use the solution of $t_{end} = 10$ to get the solution at time $t = 1$ and has been used a mesh of size $h = 0,025$ because in the previous section the solution with $h = 0,0125$ had really large computing time. The solution is shown in Figure 3:

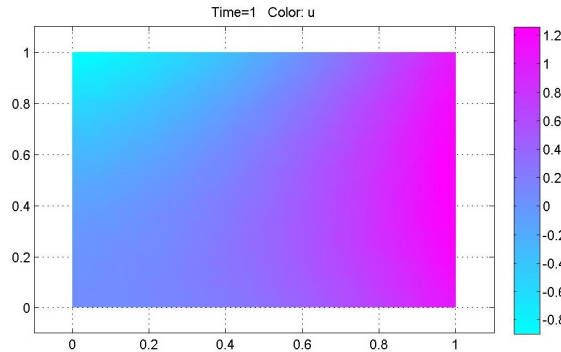


Figure 3: Temperature distribution at $t_{end} = 1$ with mesh size $h = 0,025$.

The temperature distribution is similar to Figure 1 but, as seen in the legend, are reached higher temperatures. At $t_{end} = 1$ the maximum error is $1,40 \times 10^{-3}$ and at $t_{end} = 10$ is $6,05 \times 10^{-4}$, one order of magnitude lower. The error at little times is bigger because the solution does not have enough time to convergence to the solution from the initial condition. To discuss it deeply, the error is taken for more times with time step $\Delta t = 0,01$ and is plotted in Figure 4:

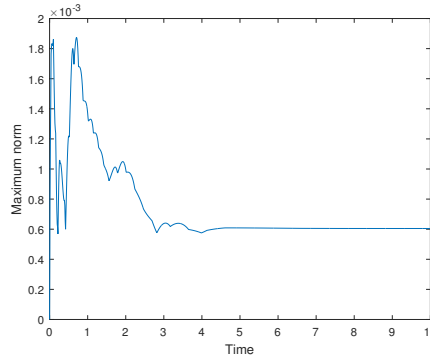


Figure 4: Maximum error vs time.

The error obtained depends on time and converges to the value at $t_{end} = 10$ with the maximum norm. This dependence is not smooth but does not change if we refine the time mesh. This is because the norm expression used in `errorExample1.m` uses the maximum function and the absolute value of the error so, maybe, the point with maximum errors is jumping through the mesh along time. Consequently, the evolution of maximum error is continuous but is not smooth. Also is observed that the error does not converge to zero, we should refine the mesh if our tolerance is lower than $6,05 \times 10^{-4}$.

3. At time $t_{end} = 50$, $e^{-50} \approx 0$ so the problem is time-independent and can be redefined as:

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega = [0, 1]^2 \\ u_n = -y & \text{in } x = 0 \\ u_n = 2 + y & \text{in } x = 1 \\ u_n = x^2 & \text{in } y = 0 \\ u_n = x - 2 & \text{in } y = 1 \end{cases}$$

Then, instead of solving a parabolic PDE, we have to solve an elliptic PDE. We have to change the PDE PDE > PDE Specification... and the boundary conditions that now does not depend on time. The analytic solution neither depends on time now. The solution of the stationary problem is the following:

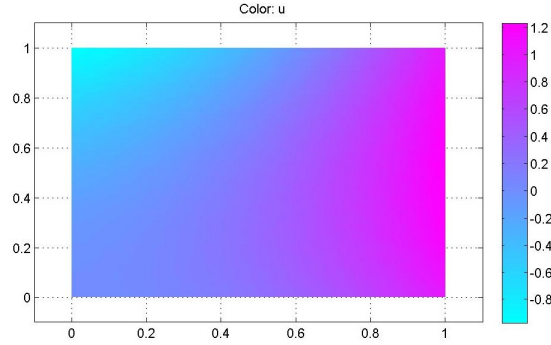


Figure 5: Stationary solution with mesh size $h = 0,025$.

While the transient problem solves an elliptic problem at each time step, the stationary problem must be computed only once, saving a lot of computational time. To compare this solution, the transient problem is also solved. In both cases a maximum error of $0,6047 \times 10^{-3}$ is obtained.