

Assignment 2: PDE-Toolbox

Due date: 2nd December

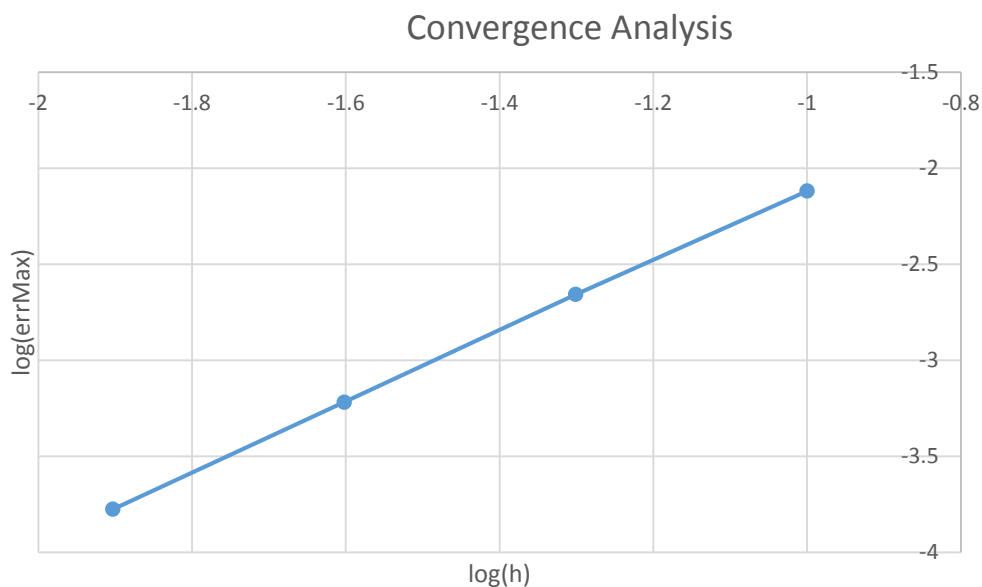
1. Solve the problem and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.

Using Matlab PDE tool we have solved the problem with the next different meshes:

- Initial mesh: 183 elements,  $h=0.1$ .
- Refined mesh: 689 elements,  $h=0.05$ .
- 2xRefined mesh: 2673 elements,  $h=0.025$ .
- 3xRefined mesh: 10529 elements,  $h=0.0125$ .

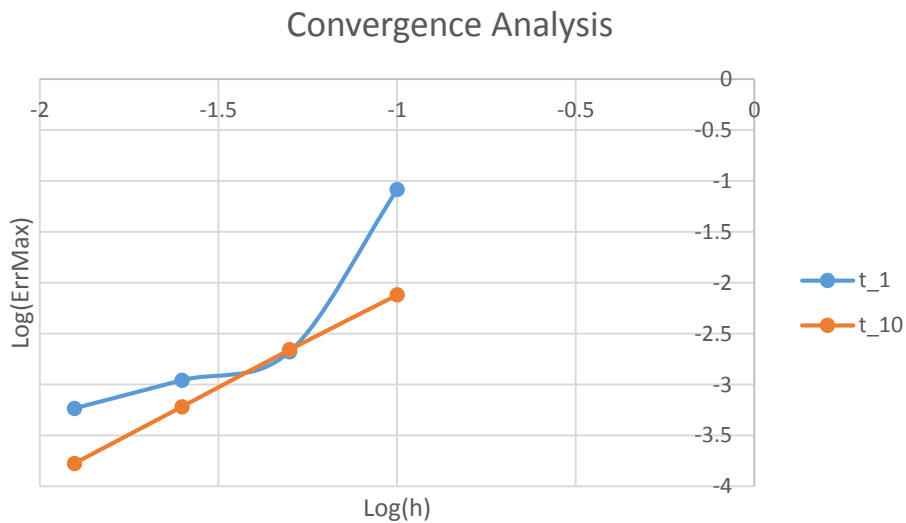
With the solutions we obtained we proceed to do a convergence analysis as seen in the example 2 from the pdetool tutorial.

Plotting the Error Max, calculated as the difference between the analytical solution and the solution for each  $h$  value we can see that the theoretical convergence order holds:



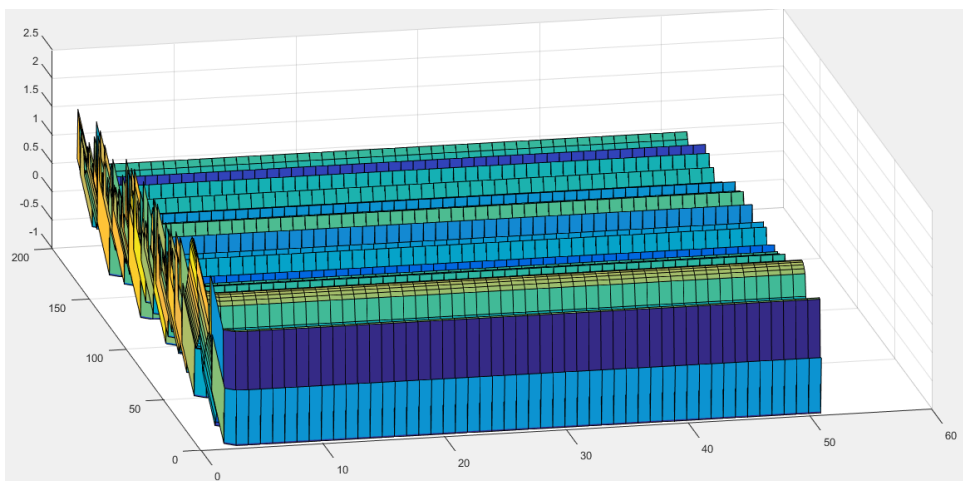
## 2. How is the solution affected when we modify the final time?

To observe how the solution is affected when we modify the time, the convergence between time equal to one and time equal to ten have been compared:



## 3. We are interested in obtaining the solution at time $t_{\text{end}} = 50$ . Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

To find a more efficient manner to solve the problem the first thing done, was to solve the partial equation with  $t=50$ . As we can see in the graphic below after a certain time, the function becomes time independent. So at time  $t=50$  the terms in the problem when time appears does not make the difference.



So, to simplify the computation the initial parabolic problem to an elliptic one:

Equation:  $-\text{div}(c^*\text{grad}(u))+a*u=f$

Type of PDE:	Coefficient	Value
<input checked="" type="radio"/> Elliptic	c	1.0
<input type="radio"/> Parabolic	a	0.0
<input type="radio"/> Hyperbolic	f	0
<input type="radio"/> Eigenmodes	d	1.0

And with the Dirichlet boundary condition re-written in the next way:

Boundary condition equation:  $h*u=r$

Condition type:	Coefficient	Value	Description
<input type="radio"/> Neumann	g	0	
<input checked="" type="radio"/> Dirichlet	q	0	
	h	1	
	r	$x.*x$	

Solving the problem with independence of the time, we simplify it and we get good results. If we compare the errors, we can see:

ErrMax_t=50	ErrMax (elliptic)
0.0076	0.0076