Polytechnic University of Catalonia

Master in Numerical Methods

Computational Mechanics Tools

Assignment 2: "PDE-Toolbox"

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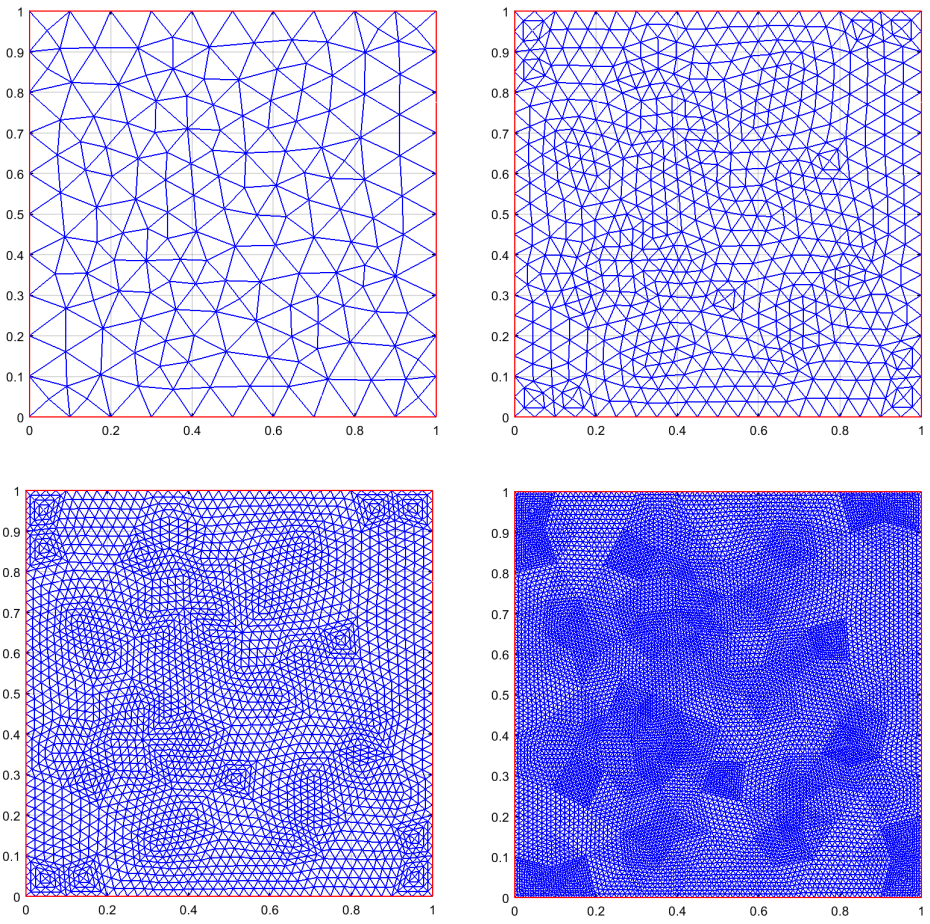
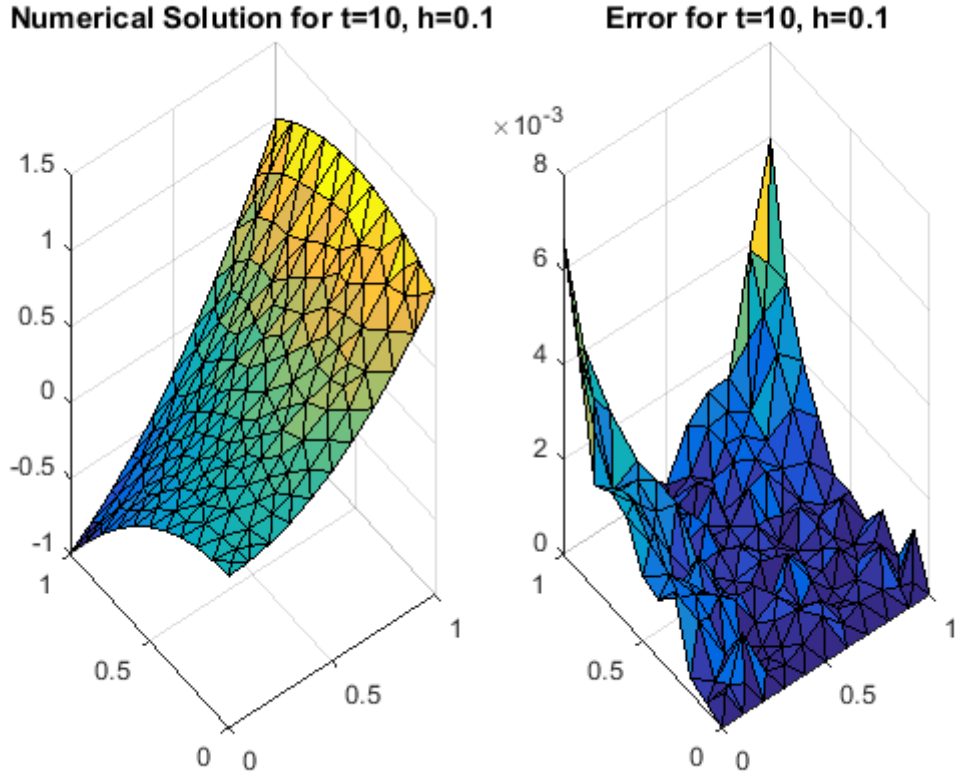
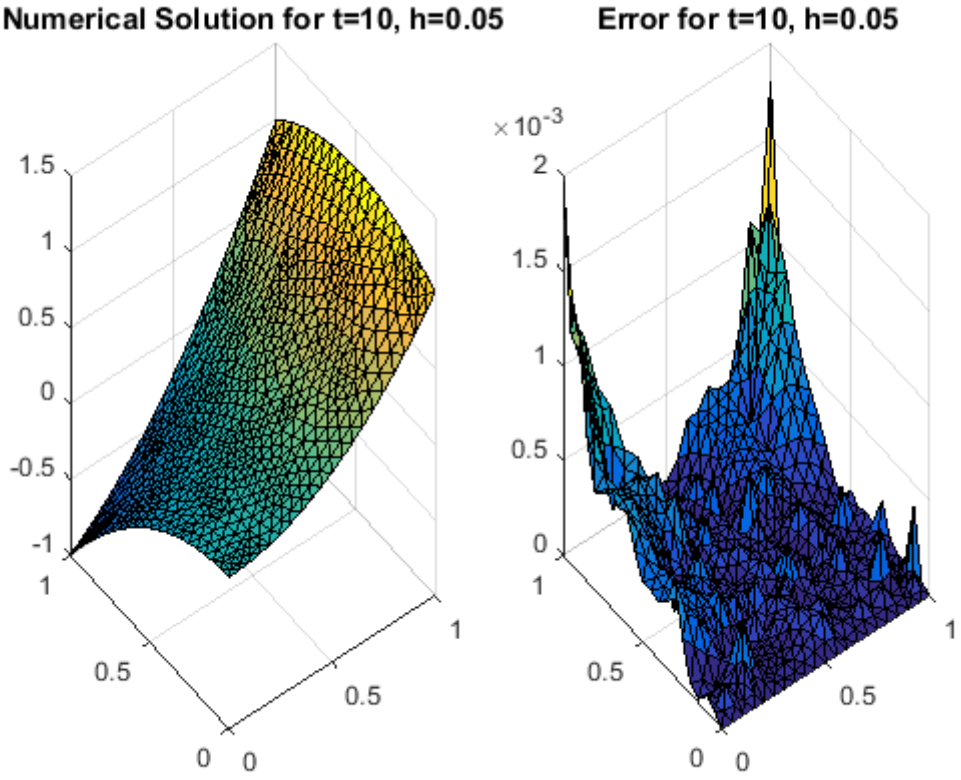
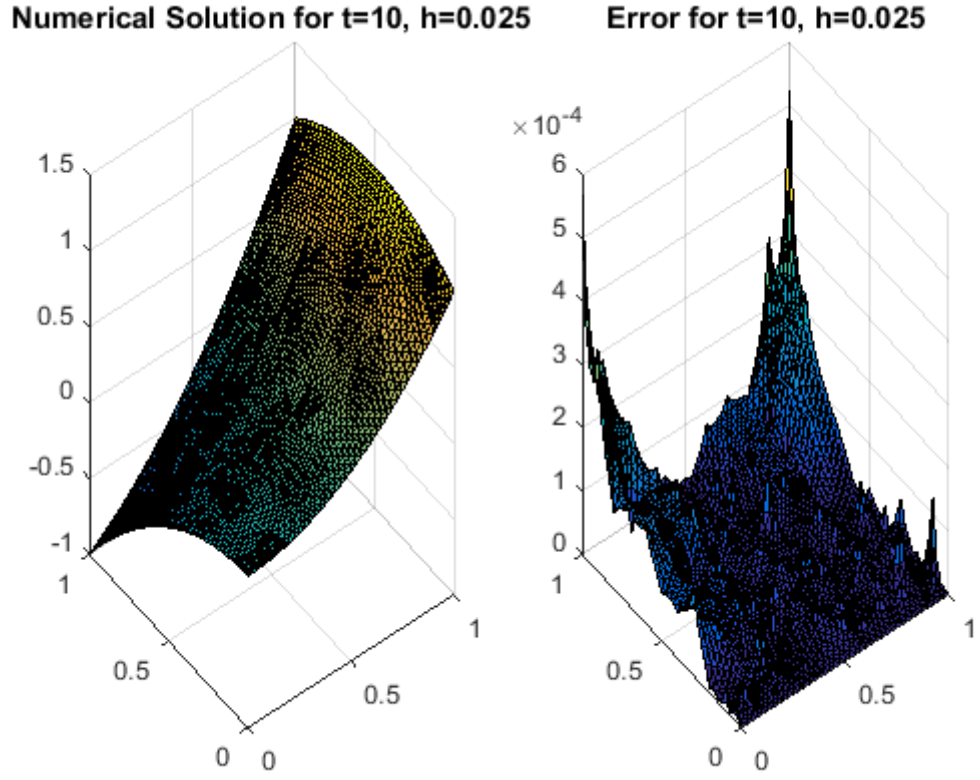
1. Solve the problem and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds

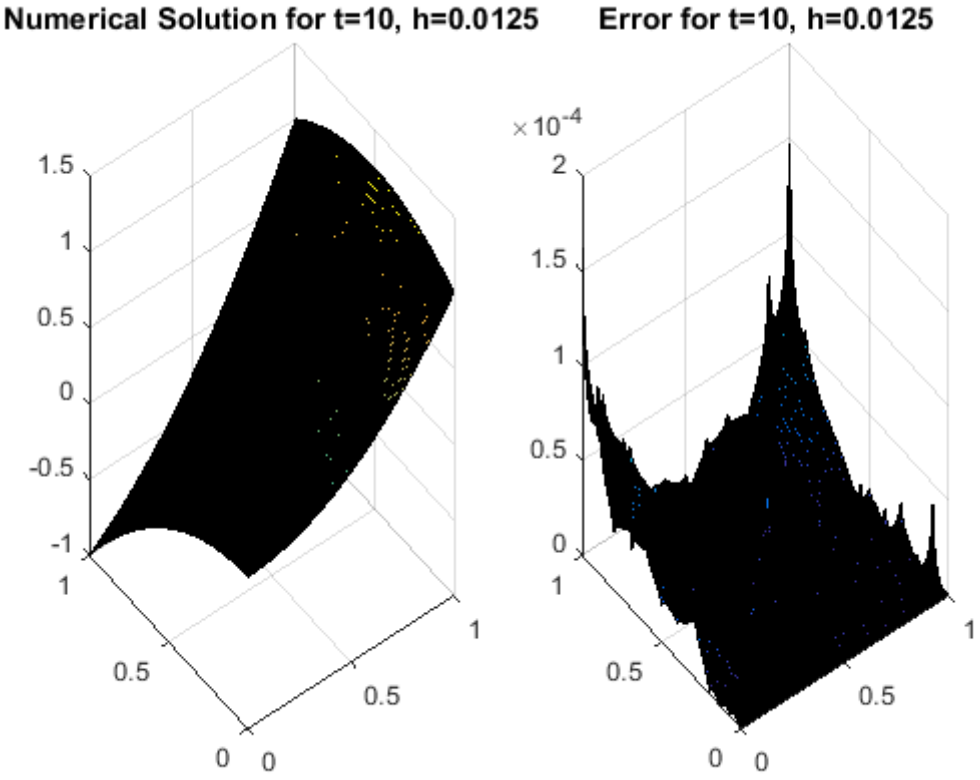


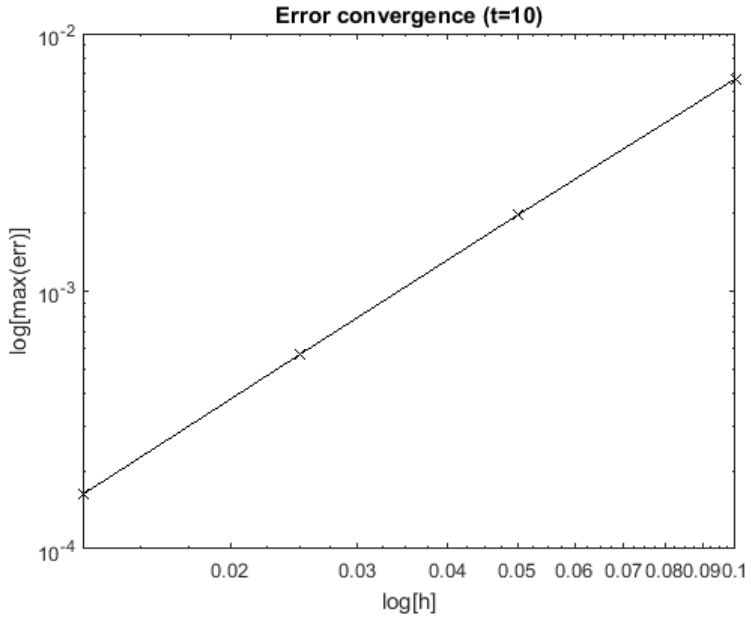
Four different meshes for the domain, with h=[0.1 0.05 0.025 0.0125]


Numerical Solution for t=10, h=0.1
Error for t=10, h=0.1
×10^-3
1.5
1
0.5
0
-0.5
-1
8
6
4
2
0
0.5
1


Numerical Solution for t=10, h=0.05
Error for t=10, h=0.05
×10^-3
1.5
1
0.5
0
-0.5
-1
2
1.5
0.5
0
0.5
1


Numerical Solution for t=10, h=0.025
Error for t=10, h=0.025
×10^-4
1.5
1
0.5
0
-0.5
-1
6
5
4
3
2
1
0
0.5
1


Numerical Solution for t=10, h=0.0125
Error for t=10, h=0.0125
×10^-4
1.5
1
0.5
0
-0.5
-1
2
1.5
1
0.5
0
0.5
1

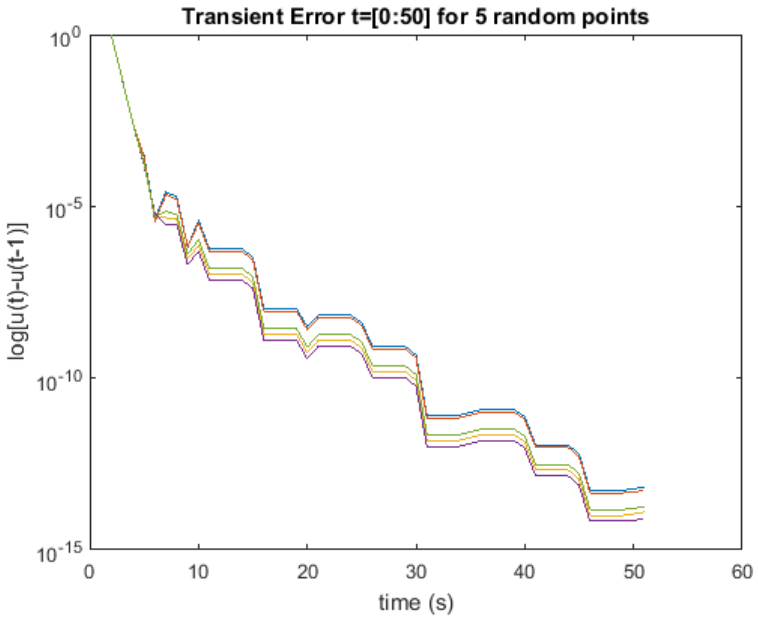


Convergence of the maximum error for the different meshes

The convergence order is given by the slope of the previous graphic (1.80)

2. How is the solution affected when we modify the final time?

The solution will change with time until it reaches a steady state. For the finest mesh, the difference between the solution for a time (t) and a time (t+1) reaches an order around 10^{-4} when $t > 5$. Taking into account that the relative and absolute tolerances used by the toolbox to solve the problem were 0.01 and 0.001 respectively, values under these quantities can be considered as computational errors.



3. We are interested in obtaining the solution at time tend = 50. Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

As the time variable appears in the equations as a negative exponential, the terms involving it tend to zero when the time is high enough. That's why instead of solving iteratively

a parabolic PDE, the problem can be solved directly by considering a time independent elliptic PDE.
For the fourth mesh, the problem was solved with this two methods, resulting in a difference between them of the order 10^{-14} which is practically null taking into account the solver's tolerance.