Computational Mechanic Tools - Assignment 2

Arthur Lustman

November 28, 2018

Part 1.

The PDETool of MAtlab is used to compute a partial differential equation and to render the solution.

$$u_t + \Delta u = f \tag{1}$$

$$f(x, y, t) = -3e^{-3t} \tag{2}$$

This equation is a parabolic PDE. Once the boundary conditions (Neumann and Dirichlet) and the final time ($t_{\text{end}} = 10$) are specified, the solution can be computed. The solution is displayed on the following figure.
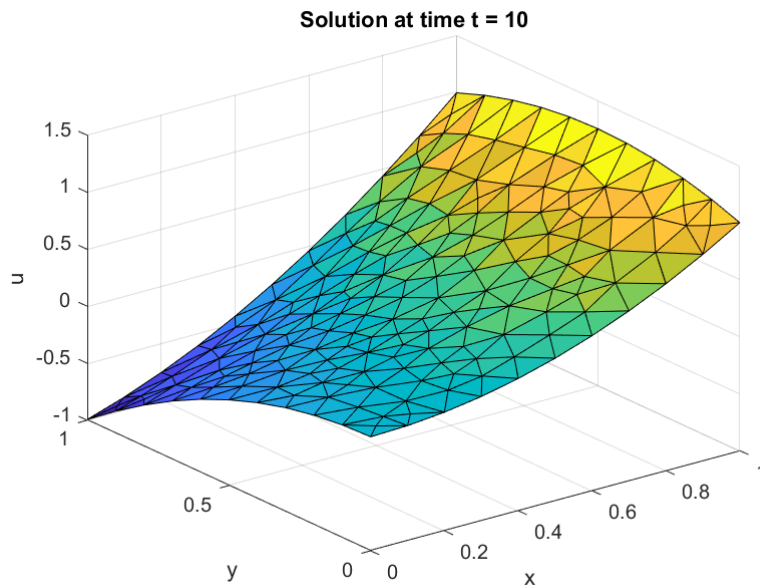

Figure 1: Solution of PDE in 3D

The solutions obtained are compared with the analytic solution, equation (3), in order to get the absolute nodal error. The maximum nodal error is saved to check the $\mathcal{L}_2$ error and the convergence order.

$$u(x, y, t) = x^2 + xy - y^2 + e^{-3t} \tag{3}$$

The mesh is refined up to 4 times from the initial mesh which gives us 5 data points. The following table gives us the values of the maximum nodal error and the element size `h`.

The element size `h` was computed using this formulae (for tetrahedral elements)

$$A = N \times \frac{h^2}{2} \tag{4}$$

Element size h	Maximum Nodal error
0.0781	0.0067
0.0390	0.0020
0.0195	5.6787e-04
0.0098	1.6004e-04
0.0049	4.4423e-05

Table 1: Maximum nodal error in order of the element size h

Knowing A, the area of the domain is equal to 1 and N to be the number of elements in the mesh given by the size of the t parameter.

The following graph is obtained and from there the convergence rate can be extracted. The variation of the slope is ≈ 1.84 which is a very good results, the theoretical result should be close to 2.
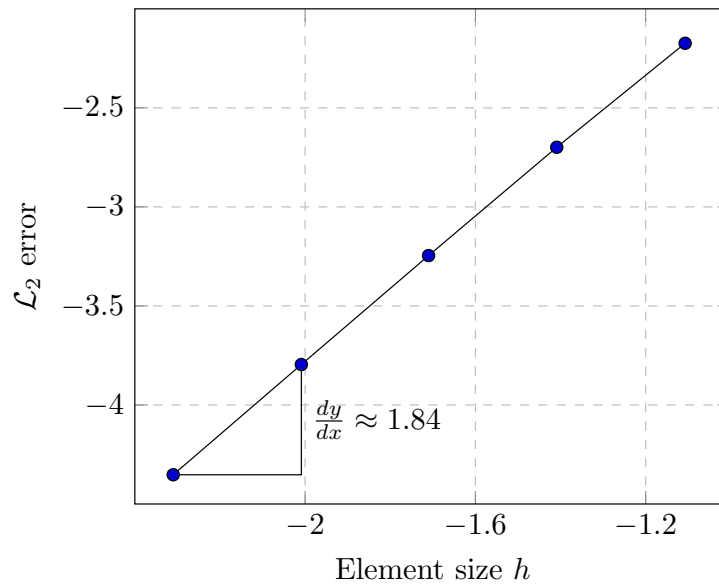
Figure 2: Convergence slope for Parabolic PDE with time $t = 10$

Part 2.

The solution is affected by modifying the final time t_{end}. The solution goes closer to a steady state equation by increasing the time t. As the time component of in the source term f decreases exponentially prior to time.

The following table represent the maximum nodal error with the same mesh parameters and 5 different time configurations t = {1,2,5,10,20}. As seen, the maximum error decreases and quickly reaches a constant value. The time doesn't significantly affect the solution (on these coarse mesh computations) after the 3rd simulation.

Time t	Maximum Nodal error
1	0.0074
2	0.0069
5	0.0066
10	0.0067
50	0.0067

Table 2: Maximum nodal error of the solution at time t

Part 3.

The solution at time $t_{\text{end}} = 50$ can be computed with the previous PDETool file which will result approximately in the same convergence slope pictured in first exercise.

A more efficient manner to compute this PDE would be to reconsider its generic form : the time parameters in this PDE are all expressed with the same coefficient e^{-3t} which convergences since $e^{-150} \approx 0$. All the time parameters are so small that they can be approximated by 0 whitout affecting the solution, which then gives us a time invariant partial differential equation.

$$\Delta u = f \tag{5}$$

$$f = e^{-150} \approx 0 \tag{6}$$

This new PDE is elliptic. The mesh is again refined up to 4 times and the solutions are compared with the analytic solution.

$$u(x, y, t) = x^2 + xy - y^2 \tag{7}$$

The computed solution with PDETool gives us the $\mathcal{L}_2$ error displayed in the following figure. This figure is very similar to the one in the first exercise but the values are slightly different, as seen on the following table. The values of the maximum nodal error are computed for the elliptic case and the parabolic case with `t=50`. These values are very close, the convergence slope is also equal to ≈ 1.85, which suggest both method are numerically equivalent.

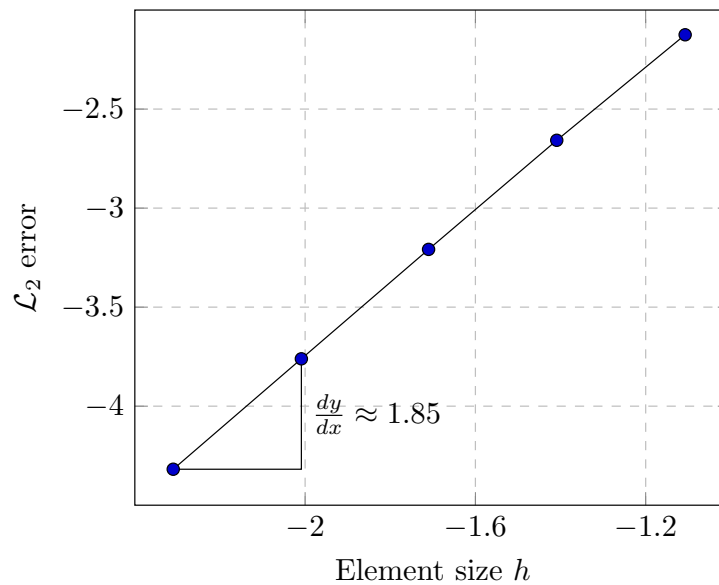| Parabolic's solution error	Elliptic's solution error
0.0067	0.0075
0.0020	0.0022
5.6788e-04	6.1882e-04
1.5992e-04	1.7344e-04
4.4414e-05	4.8039e-05

Table 3: Comparison of the maximum nodal error

Figure 3: Convergence slope for Elliptic PDE