UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER IN COMPUTATION MECHANICS AND NUMERICAL METHODS IN ENGINEERING

COMPUTATIONAL MECHANICS TOOLS

ASSIGNMENT - PDEtool

by

Renan Alessio

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1- Introduction

The goal of the assignment is to solve the following problem using the MatLab PDE Toolbox feature:

$$u_t - \Delta u = f \qquad in\ \Omega = [0,1]^2 \tag{1}$$

Where f(x,y,t) is defined as :

$$f(x,y,t) = -3e^{-t}$$

with the followig initial conditio t=0 :

$$u(x,y,t=0) = x^2 + xy - y^2 + 1$$

with the following boundary conditions :

$$u_n(x=0,y,t) = -y$$

$$u_n(x=1,y,t) = 2 + y$$

$$u_n(x,y=0,t) = x^2 + e^{-3t}$$

$$u_n(x,y=1,t) = x - 2$$

2 – Solving the equation for t=10 and evaluating the convergence

Figure 1 depicts the solution of the problem considering t=10 and an initial mesh with average element size h of 0.0781. It is worth mentioning that the average element size was calculated considering that each element has the same area and that the elements are right triangles with legs of same size.

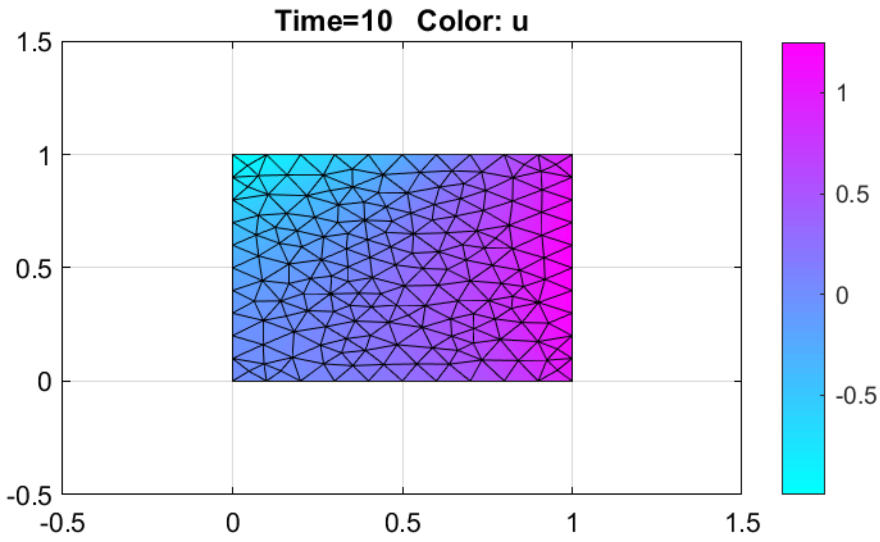

Figure 1. Solution for t=10 with an initial mesh of average element size h of 0.078.

To evaluate the solution convergence, the mesh was refined four times. Table 1 presents the number of elements, the values of the average element size h and the maximum absolute error between the analytical numerical solution for each considered mesh.

Table 1. Mesh parameters

Mesh	Number of elements	Average element size h	Maximum absolute error
1 (initial)	328	0.078	0.0067
2	1312	0.039	0.0020
3	5248	0.0195	0.0005687
4	20992	0.0098	0.00016004
5	83968	0.0049	0.000044423

Taking into account the data presented in Table 1, it is possible to plot the convergence of the solution in log scale considering the maximum absolute error on the y-axis and the average element size h on the x-axis. Figure 2 depicts the solution convergence.

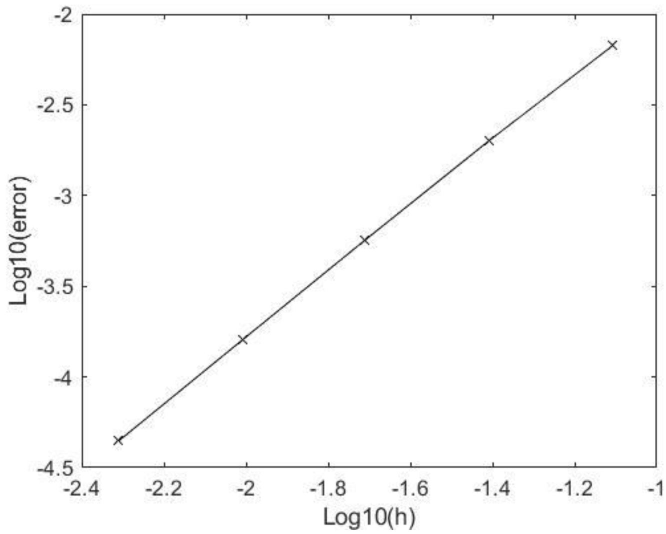

Figure 2. Solution convergence considering the maximum absolute error and the average element size h.

According to Figure 2, as the average element size h decreases, the maximum absolute error also decreases in a linear way. Such behavior was expected since the domain is discretized every time with more elements. Also, the slope of the curve presented in Figure 2 is equal to 1.8491. With the linear behavior of the solution convergence and such value for its slope, the theoretical convergence order holds.

3 – Solution evaluation with different final times

To evaluate how the solution behaves with time, the solutions for time equals to 1 and 10 are presented in Figure 3. The value of the function u(x,y,t) was also evaluated at the point of its maximum absolute value with time varying from 0 to 30. It worth mentioning that such evaluation was made with the mesh with average element size h of 0.078. Figure 3 depicts how the maximum value of u(x,y,t) changes with time varying from 0 to 30.

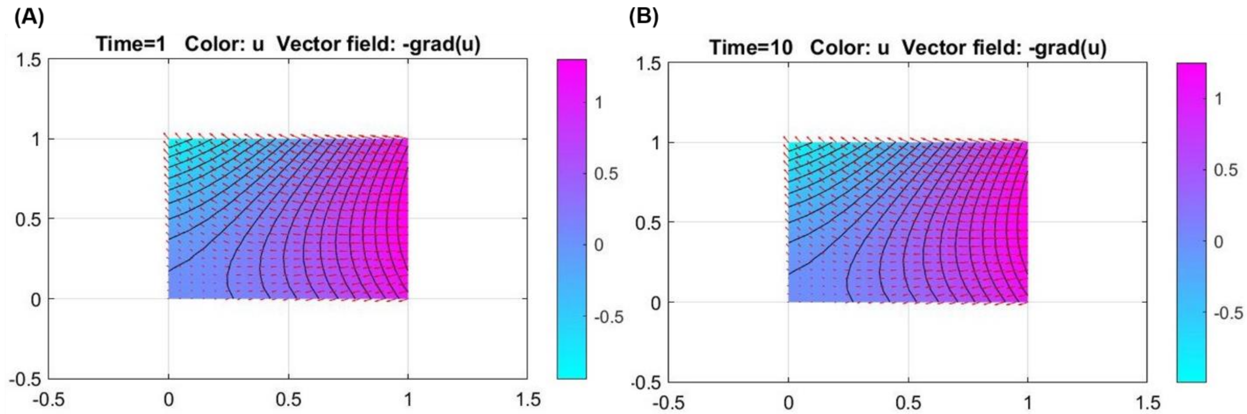

Figure 3. Solutions for time equals 1 (A) and time equals 10 (B)

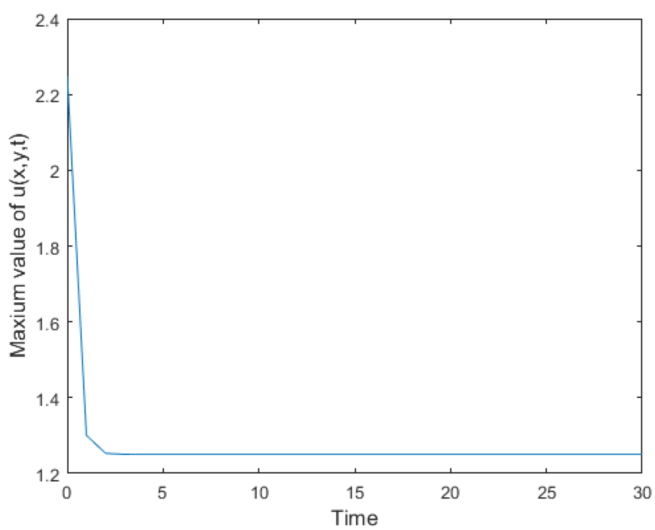

Figure 4. Change in maximum absolute value of u(x,y,t) with time varying from 0 to 30

The solutions depicted in Figure 3 have similar behavior, indicating a small variation of the solution between the times 1 and 10. According to Figure 4, the maximum value of u(x,y,t) reaches an almost steady state (small variation as time increases) before time t equals 5. Such behavior is expected since the term in the analytical solution which is related to time tends to zero as time increases. It is possible to say that the transient term in the solution is negligible after time t reaches a certain value.

4 – Solution for t=50 and a more efficient way to solve the solution

The solution of the stated problem for time = 50 is presented in Figure 5. For this solution, a mesh of average element size h of 0.078 was applied.

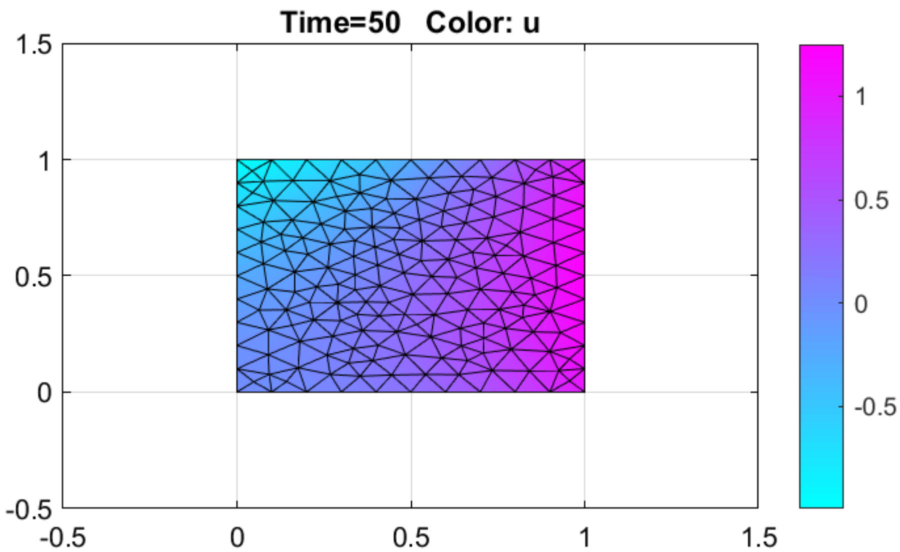

Figure 5. Solution for time t equals 50 with average element size h of 0.078.

Since it was observed in Figure 3 that the transient term in the analytical solution becomes negligible after a certain time t, a new approach to solve the stated problem (solution for time = 50) is presented. As the terms with time dependence tend to zero as time increases, the new approach of solving the problem would be to transform Equation (1) into an elliptic partial differential equation instead of remaining as a parabolic partial differential equation. All the time dependent terms would be crossed out of the equation, making it simpler to solve. Equation (1) would be rewritten as follows with the following boundary conditions:

$$-\Delta u = 0 \qquad in\ \Omega = [0,1]^2 \tag{2}$$

with the following boundary conditions :

$$u_n(x = 0, y) = -y$$

$$u_n(x = 1, y) = 2 + y$$

$$u_n(x, y = 0) = x^2$$

$$u_n(x, y = 1) = x - 2$$

The solution of Equation 2 with a mesh of average element size h of 0.078 is presented in Figure 6. A comparison between the counter plots of solution of Equation (1) for time = 50 and solution of Equation (2) is depicted in Figure 7.

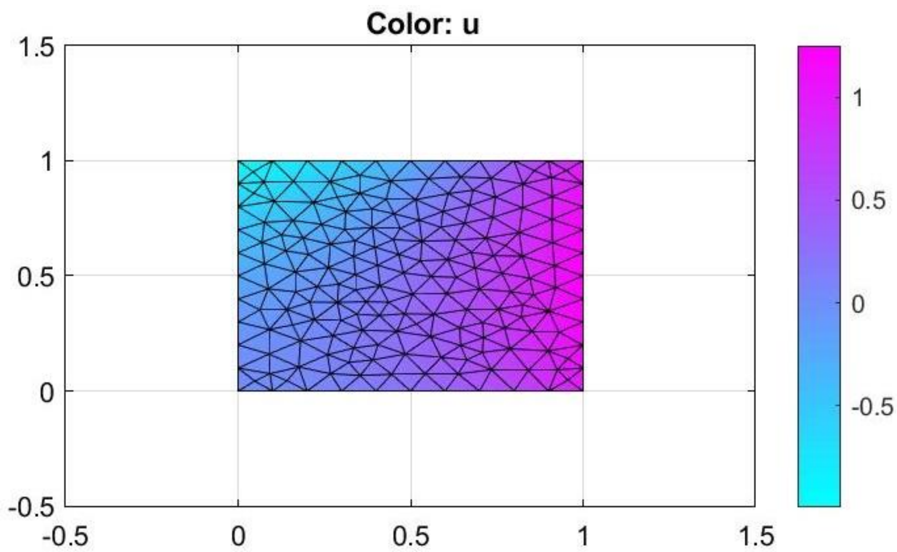

Figure 6. Solution of Equation (2).

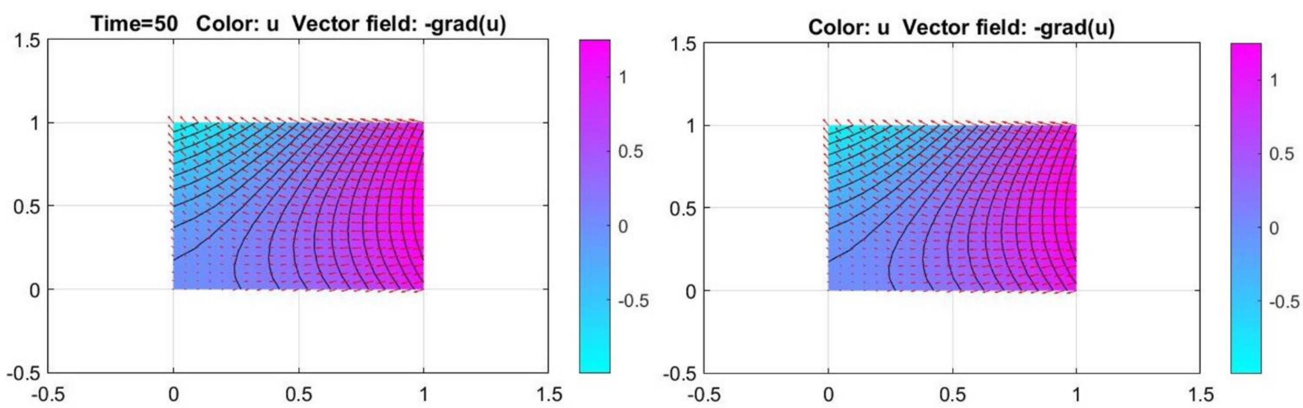

Figure 7. Comparison between counter plots of solutions from Equation (1) for time = 50 (left hand counter plot) and Equation (2) (right hand counter plot).

According to Figure 7, the solution of Equation (2) represents well the solution of Equation (1) for time = 50 since the counter plots present similar behavior. The maximum value of u when solving Equation (1) is 1.250327219869620 and Equation (2) is 1.250327219869678, presenting a relative difference of order 10^{-14}. Also, the CPU time reduced from 863.2969 to 829.7344 when solving Equation (2) instead of Equation (1) for the stated problem. Since Equation (2) presents an accurate solution (14 equal digits) in comparison with solution of Equation (1) for time =50 and there was a reduction in the CPU time, solving Equation (2) can be considered as more efficient to obtain the solution for Equation (1) with time = 50.