Computational Mechanics Tools

PDE Toolbox

Assignment 3

Implementation of PDE Toolbox to solve Parabolic PDE

Shardool Kulkarni

December 13, 2019

UNIVERSITAT POLITÈCNICA DE CATALUNYA

1 Introduction

The problem given is as follows, on the domain $\Omega = [0, 1]^2$

$$u_t - \Delta u = f \tag{1}$$

$$f(x, y, t) = -3e^{-3t} \tag{2}$$

The initial condition is as follows

$$u(x, y, t = 0) = x^2 + xy - y^2 + 1 \tag{3}$$

The boundary conditions are as follows :

$$u_n(x = 0, y, t) = -y \tag{4}$$

$$u_n(x = 1, y, t) = 2 + y \tag{5}$$

$$u(x, y = 0, t) = x^2 + e^{-3t} \tag{6}$$

$$u_n(x, y = 1, t) = x - 2 \tag{7}$$

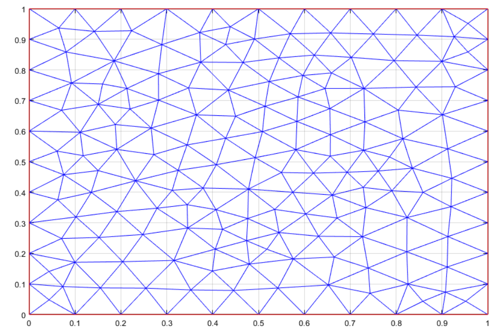
here $u_n = \frac{\partial u}{\partial n}$
The equation is Parabolic in nature and is solved is of the form in the PDEtoolbox

$$d\frac{\partial u}{\partial t} - \nabla.c(\nabla u) + au = f \tag{8}$$

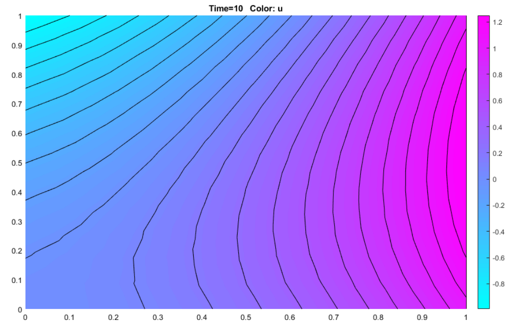
with $d = 1$, $c = 1$ $au = 0$ and $f = -3e^{-3t}$

2 Results

The solution for $t = 10$ case is presented in the following contour plot.

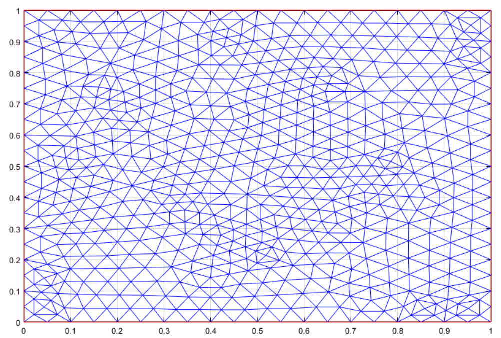


(a) Mesh

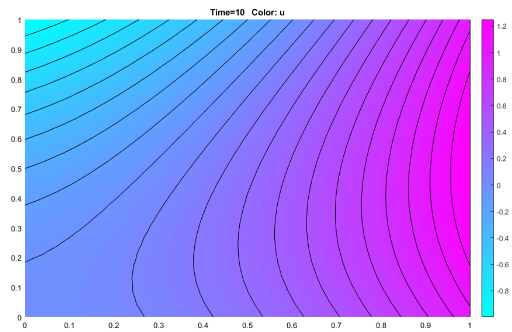


(b) Contour

Figure 1: Initial Contour and Mesh

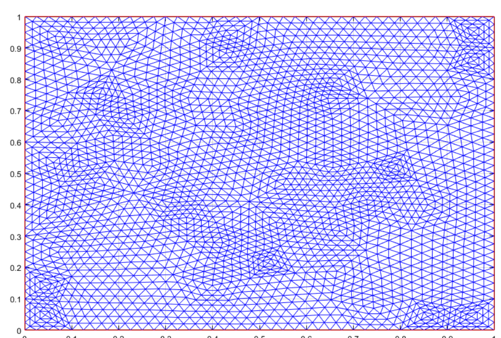


(a) Mesh

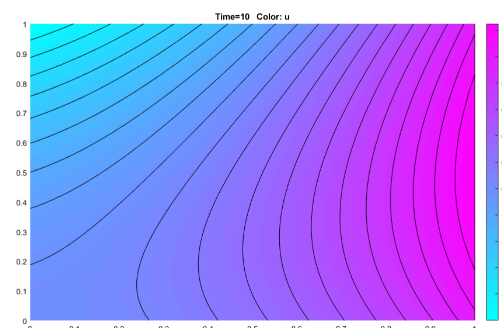


(b) Contour

Figure 2: Contour and Mesh after first refinement

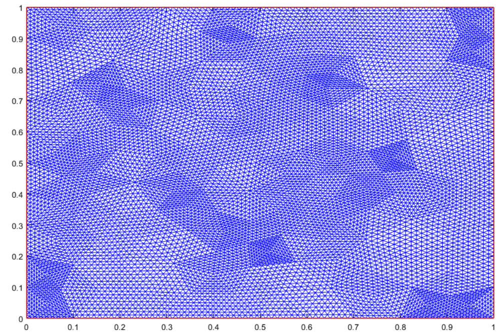


(a) Mesh

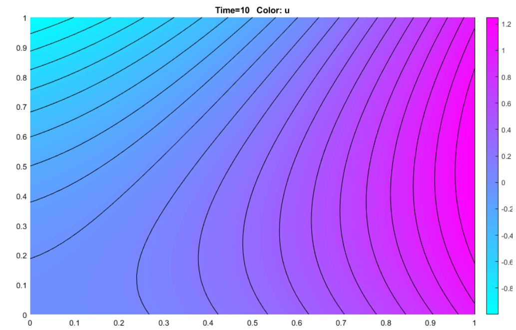


(b) Contour

Figure 3: Contour and Mesh after second refinement

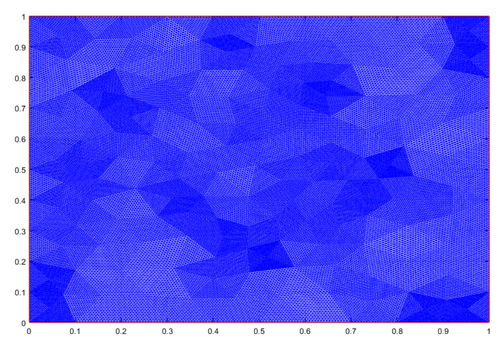


(a) Mesh

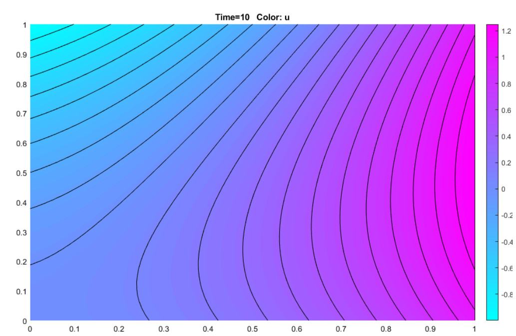


(b) Contour

Figure 4: Contour and Mesh after third refinement



(a) Mesh



(b) Contour

Figure 5: Contour and Mesh after fourth refinement

3 Error Plot

To check for the convergence we find look at the variation of max error as we refine the mesh successively. The average element size h is computed from the following relation. Where the N is the number of nodes, and A is the area of the square domain.

$$h = \sqrt{\frac{2A}{N}} \tag{9}$$

The errors for the time period t = 10 are presented below

Mesh	Number of Nodes	Average Element Size	Error
Initial	312	0.080064	0.0075
First refinement	1248	0.040032	0.00222
Second refinement	4992	0.020016	0.0006186
Third refinement	19968	0.010008	0.0001739
Fourth refinement	79821	0.005004	0.00004778

Figure 6: Table of element size and error for t=10

The errors for the time period t = 1 are presented below

Mesh	Number of Nodes	Average Element Size	Error
Initial	312	0.080064	0.0848
First refinement	1248	0.040032	0.0804
Second refinement	4992	0.020016	0.0791
Third refinement	19968	0.010008	0.0787
Fourth refinement	79821	0.005004	0.0786

Figure 7: Table of element size and error for t=1

The slopes of the two curves are given below

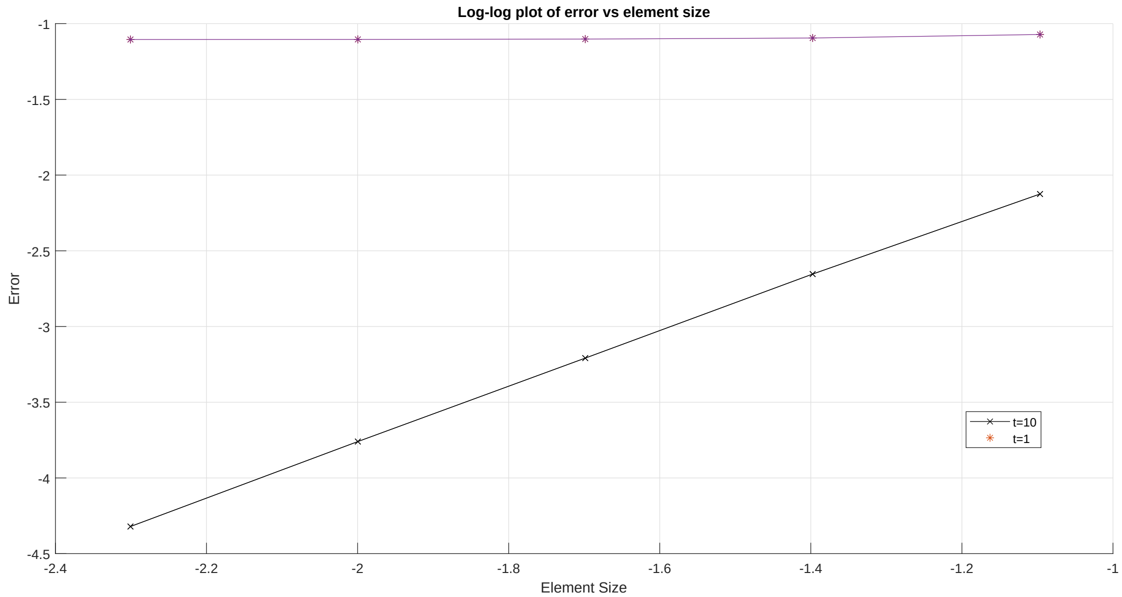


Figure 8: Log-log plot of error vs time interval for t=1 and t=10

Curve	Slope	Linear Fit equation
t = 10	1.83	1.83x − 0.111
t = 1	0.025	0.025x − 1.05

Figure 9: table of slopes

4 T=50 secs

The errors obtained for t = 10 are almost constant after time $t = 4$ secs, we can say that the system has reached a state at that point. The graph of the error variation in figure 10 illustrates it. Therefore the errors for $t = 50$ secs would be the same as that for time $t = 10$ secs. This could be because of the fact the exponential function on the RHS of the PDE is also a rapidly decreasing function.

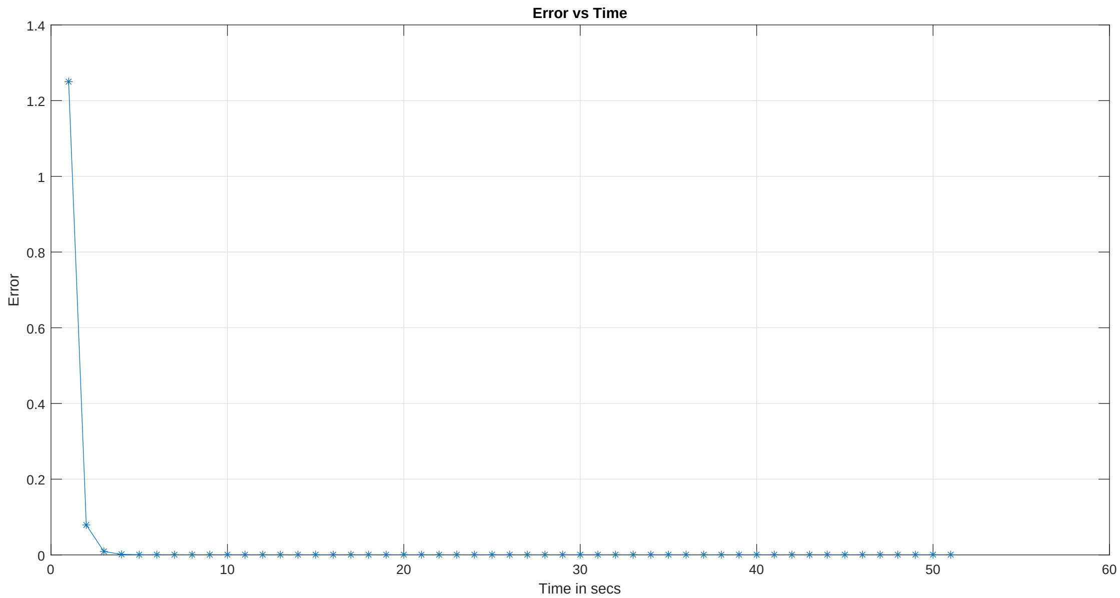


Figure 10: Variation of error