UNIVERSITAT POLITÈCNICA
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BARCELONATECH

CIMNE

Master in Computational Mechanics

Computational Mechanics Tools

Assignment 1:
PDE-Toolbox

Year: 2019-20

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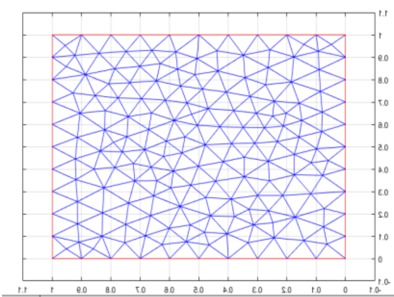
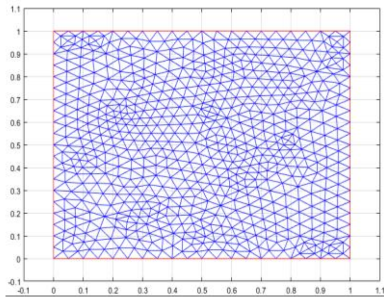
1. **Consider t_{end} = 10, solve the problem, and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.**

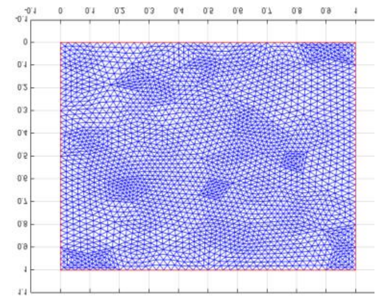
In order to plot the convergence plot between $\log_{10}h$ vs $\log_{10}err$ (err = array containing max. error value in each mesh), we need to refine the initial mesh four times as mentioned.



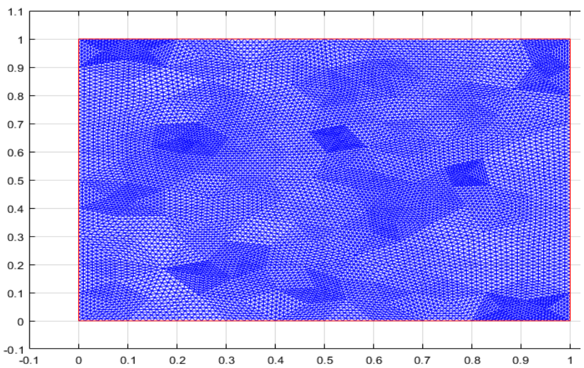
Initial Mesh (183 Nodes and 324 triangles)



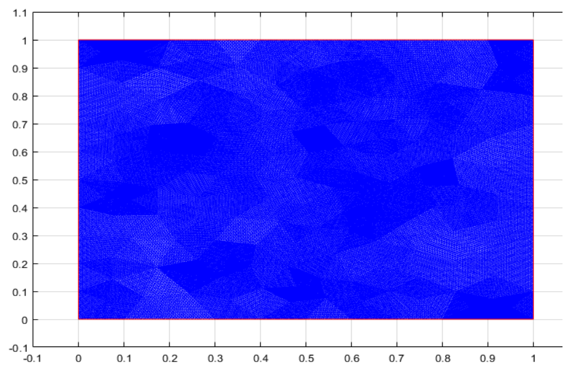
1st Mesh refinement (689 Nodes and 1296 triangles)



2th Mesh refinement (2673 Nodes and 5184 triangles)



3rd Mesh refinement (10529 Nodes and 20736 triangles)



4th Mesh refinement (41793 Nodes and 82944 triangles)

Figure 1: Initial Mesh and its 4 times consecutive mesh refinements

Now, first h is computed using the formula:

Mesh size, $\boldsymbol{h} = \sqrt{2.A}/N$, where, A = Area of the domain & N = number of elements

	Between h1 & h2	Between h2 & h3	Between h3 & h4	Between h4 & h5
Slope	1.8191	1.8365	1.8509	1.8583

Table1: Values of slopes for t_{end}=10 sec

	Initial Mesh	1st time Mesh Refinement	1st time Mesh Refinement	1st time Mesh Refinement	4th Mesh refinement
Max. value of Error (**ErrMax**)	0.0076	0.0022	6.0532e^{-04}	1.6781e-^{05}	4.6282e-^{05}

Table 2: Maximum Values of Errors at initial mesh & its consecutive refinements

The average slope of graph is $\log_{10}h$ Vs $\log_{10}Errmax$ is 1.84. So, the convergence plot holds good at t = 10 sec.

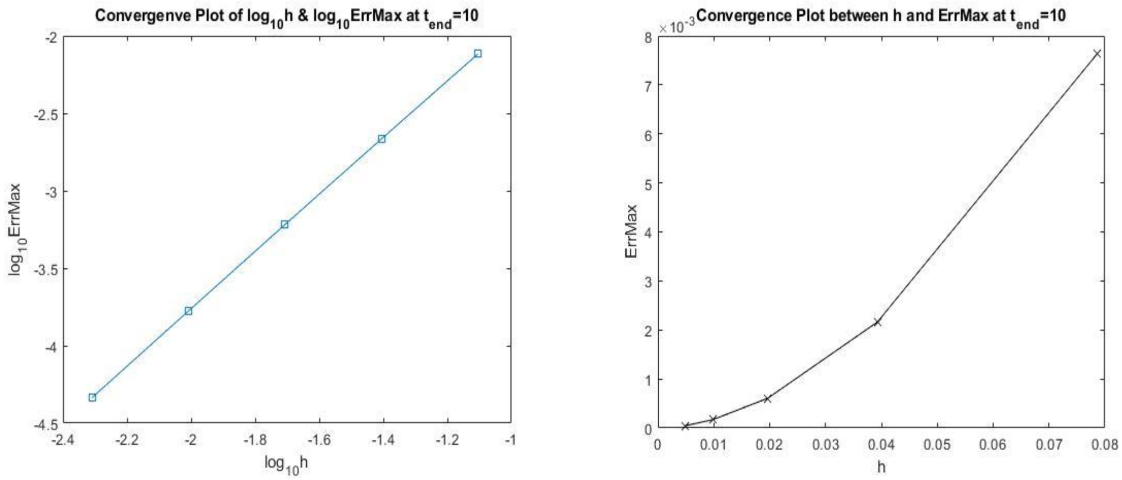


Figure2: Convergence plots of $\log_{10}h$ Vs $\log_{10}$ (Max. Error value) & h Vs Max. Error value

2. **How is the solution affected when we modify the final time?**

When the final time is modified the final solution is affected. To show this, we will be comparing the solution at t=0 sec with that at t=10 sec. The convergence plots for t_{end}=1 sec & t_{end} =10 are shown below. Convergence plot follows a non-linear decreasing pattern for each mesh refinement. While for t_{end}=10 sec, the pattern is linear.

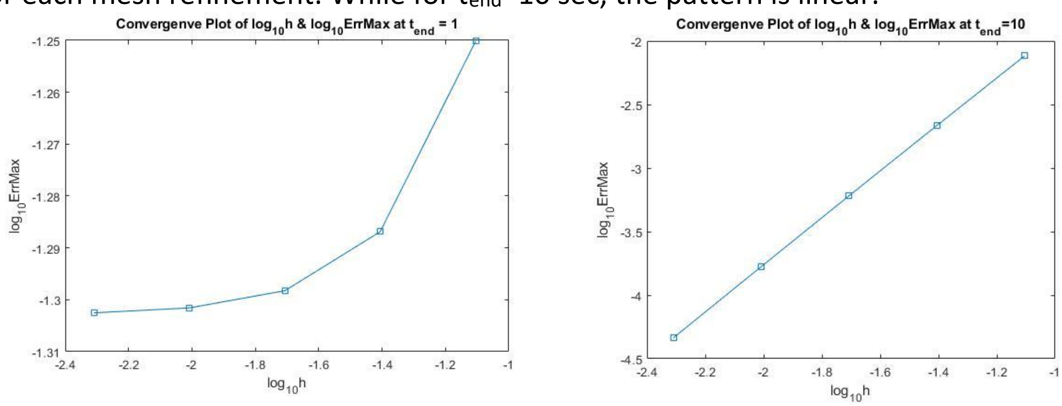


Figure 3: Convergence plots of $\log_{10}h$ Vs $\log_{10}$ (Max. Error value) at t_{end}=1 (Left) &t_{end}=10 (Right)

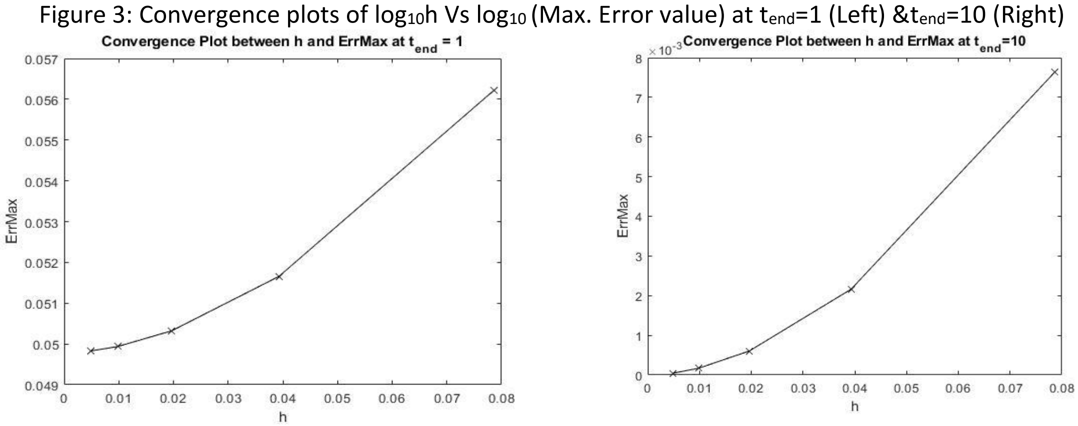


Figure 4: Convergence plots of Mesh size(h) Vs (Max. Error value) at t_{end}=1 (Left) &t_{end}=10 (Right)

In other words, it can be stated that after some time the given equation becomes time-independent.

3. **We are interested in obtaining the solution at time tend = 50. Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.**

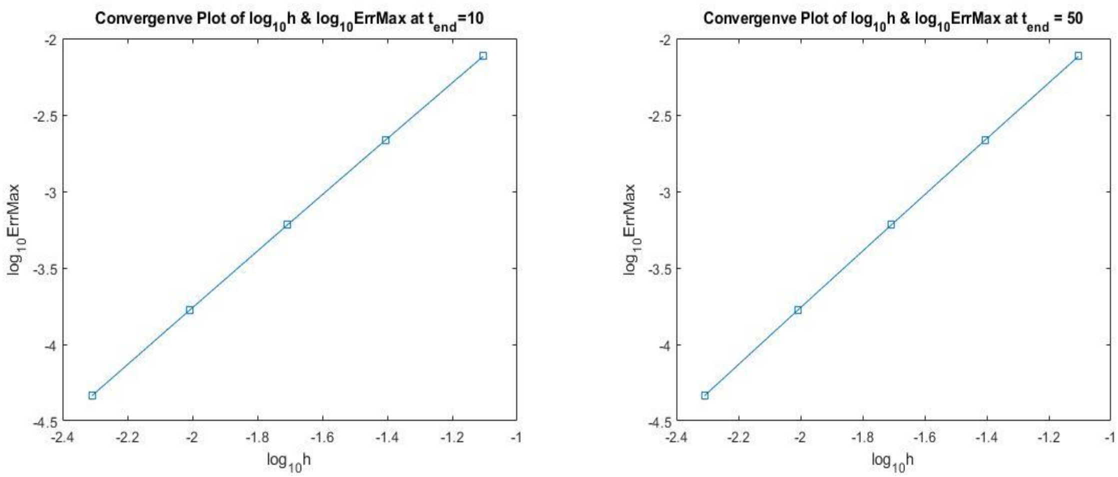


Figure 5: Convergence Plots b/w $\log_{10}h$ Vs $\log_{10}$(MaxErr) at t_{end}=50 (left) & at t_{end}=50(right)

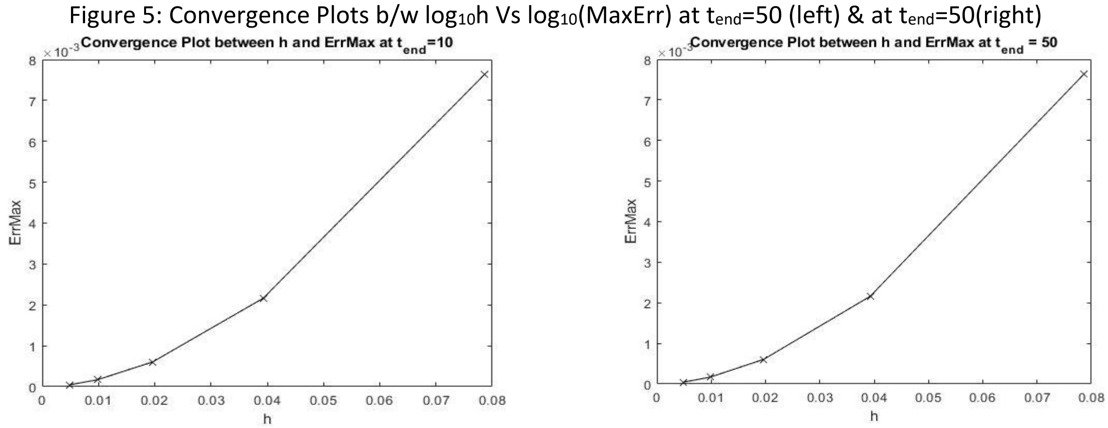


Figure 6: Convergence Plot of Mesh size(h) Vs Max. Error Value (t_{end}=10)(left) & h Vs Max. Error Value(t_{end}=50)(right)

It can be observed from the graphs that the slopes at time t=10 sec and t=50 sec are almost the same, which signifies that after some times the equation becomes independent of t.

For this purpose, two time (t) Vs u (numerical solution) graphs are plotted at t_{end}=10 sec & t_{end}=50 sec as shown below.
Both graphs illustrates that after t=2 seconds, equation is independent of time. This indicates that when t_{end} = 50 sec, the given can be considered as an elliptic equation. Therefore, when this equation becomes time-independent, then it can be considered as elliptical equation to avoid more computations.

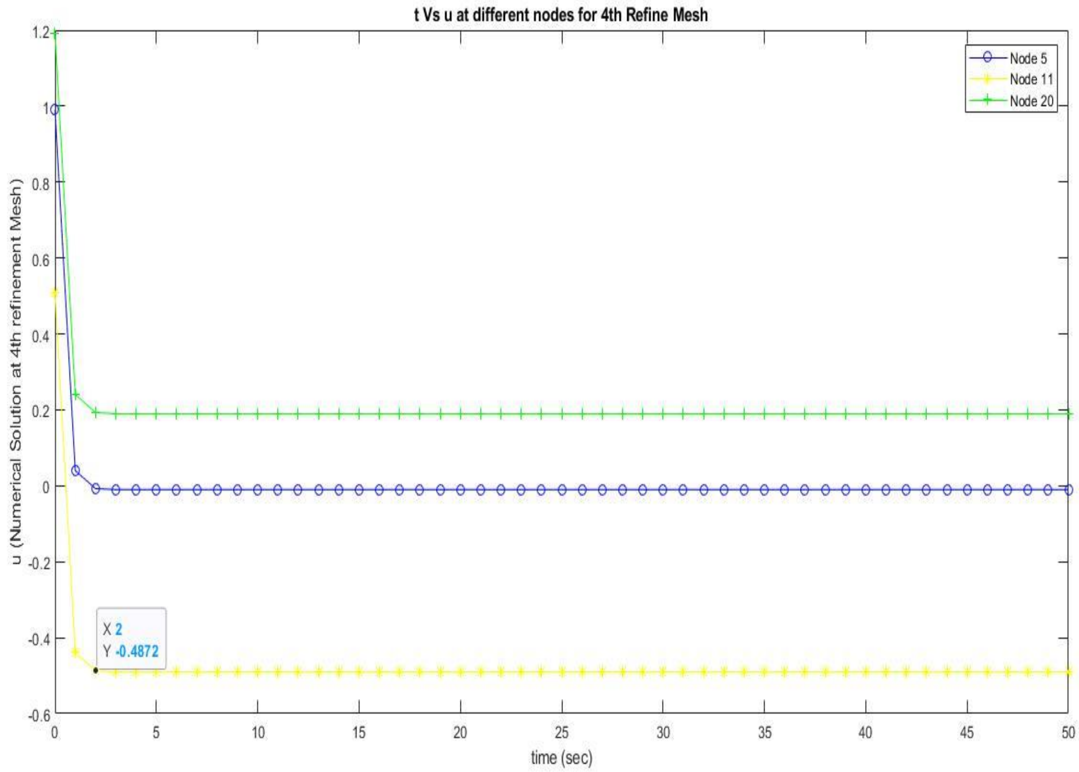


Figure 7: Numerical solution Vs time plot at random nodes for last Mesh at t_{end} = 50 sec

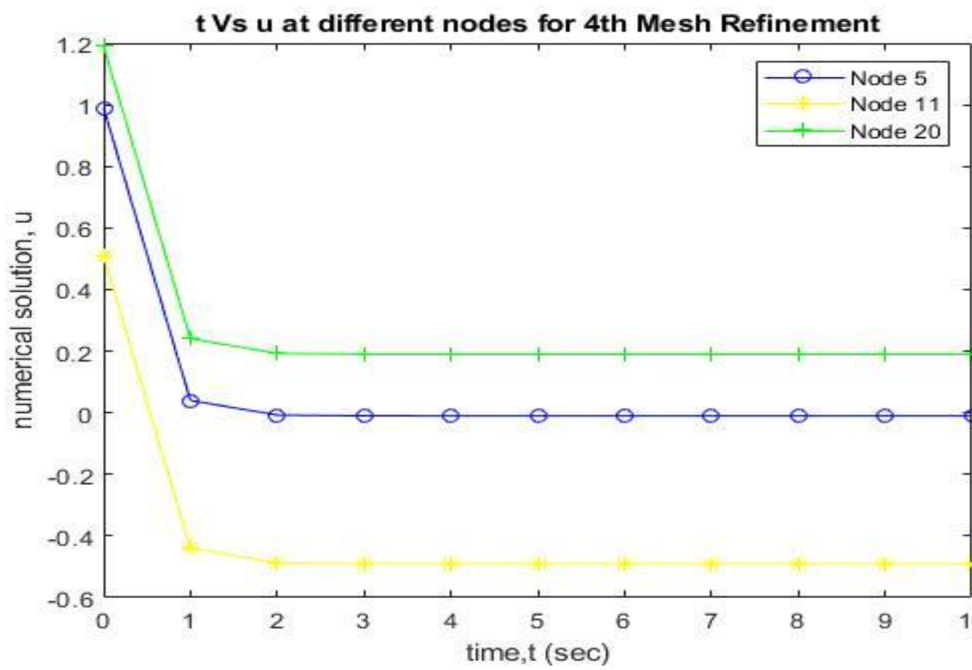


Figure 8: Numerical solution Vs time plot at random nodes for last Mesh at t_{end} = 10 sec