

Solve the following problem with the MATLAB PDE Toolbox:

$$u_t - \Delta u = f \quad \text{in } \Omega = [0, 1]^2,$$

where the source term is given by,

$$f(x, y, t) = -3e^{-3t}.$$

We consider an initial condition at  $t = 0$ :

$$u(x, y, t = 0) = x^2 + xy - y^2 + 1,$$

and the following boundary conditions

$$u_n(x = 0, y, t) = -y,$$

$$u_n(x = 1, y, t) = 2 + y,$$

$$u(x, y = 0, t) = x^2 + e^{-3t},$$

$$u_n(x, y = 1, t) = x - 2,$$

where  $u_n \equiv \partial u / \partial n$ .

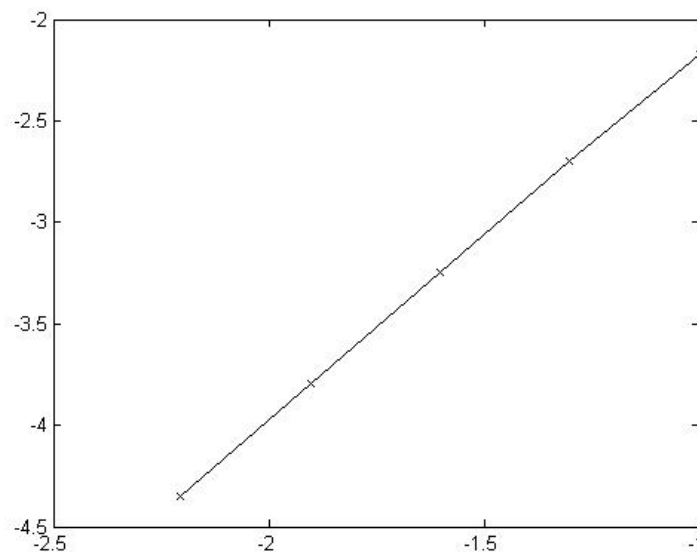
1. Solve the problem and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.

We have to solve a parabolic problem that is time dependent with some Dirichlet and Neumann boundary conditions and initial conditions.

As we are told we have to refine the initial mesh four times, after doing these we can compute the error for each refining comparing the numerical solution computed with Matlab and the analytical solution given by the following expression:

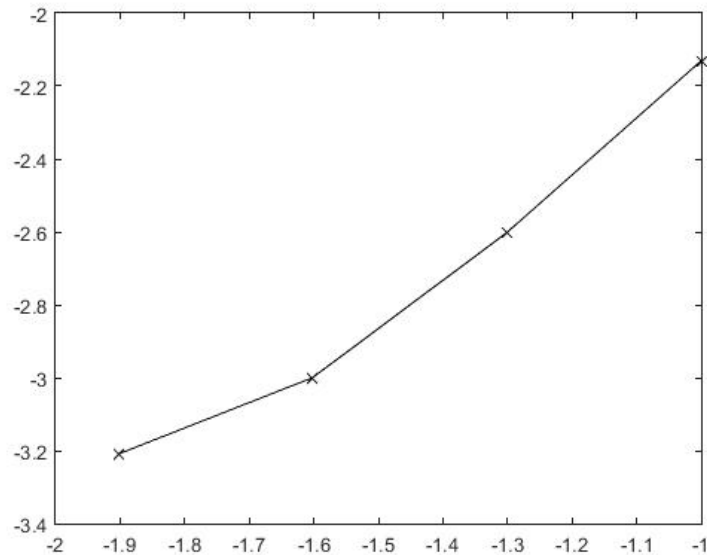
$$u(x, y, t) = x^2 + xy - y^2 + e^{-3t}.$$

We get the value of the error for the four refining and we can represent the solution in a plot as we can see. These plot shows that the theoretical convergence order holds.



2. How is the solution affected when we modify the final time?

Here we can see the representation of the error when our initial time is 1. We can observe that initially the solution behaves in a transitory way and after a while gets stationary. This effect is reflected when we use bigger values of  $t$ , as we have seen in the first plot showing the convergence of the error for time 10.



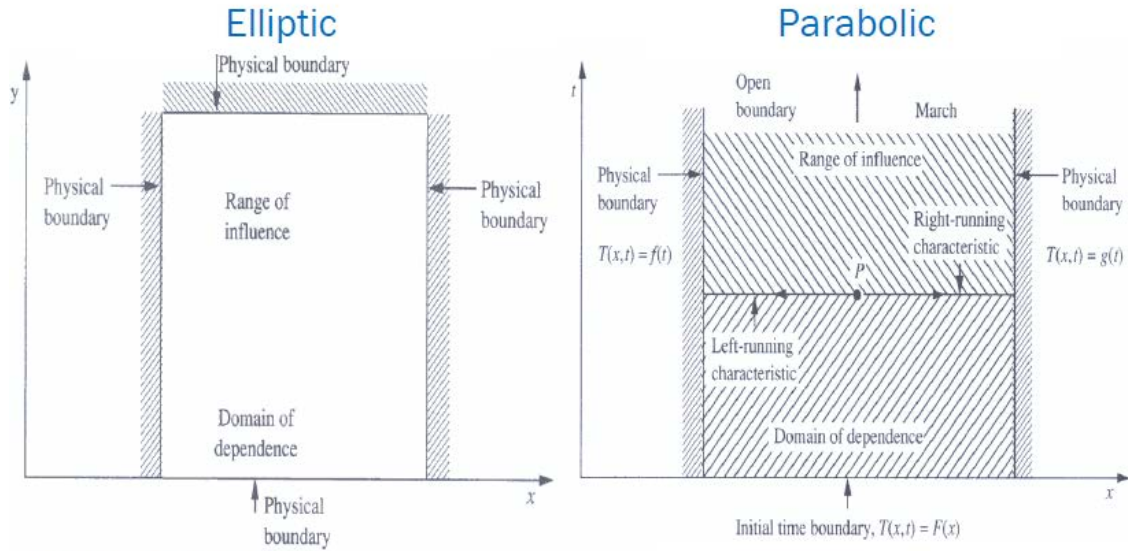
3. We are interested in obtaining the solution at time  $t_{\text{end}}=50$ . Find a more efficient manner to solve this problem. You don't need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

These types of problems have two parts: transitory and stationary part. As we can see for  $t=50$  we are in the stationary part of the problem, so it is non time dependent. A more efficient way to solve this problem would be considering that is an elliptic problem. To solve the problem we will not need initial boundary conditions and all the boundary conditions and functions will be non time dependent.

We can deduce that for big values of time the problem gets stationary comparing the values of the error for  $t=10$  and  $t=50$  that they get really close to small values for the parabolic problem.

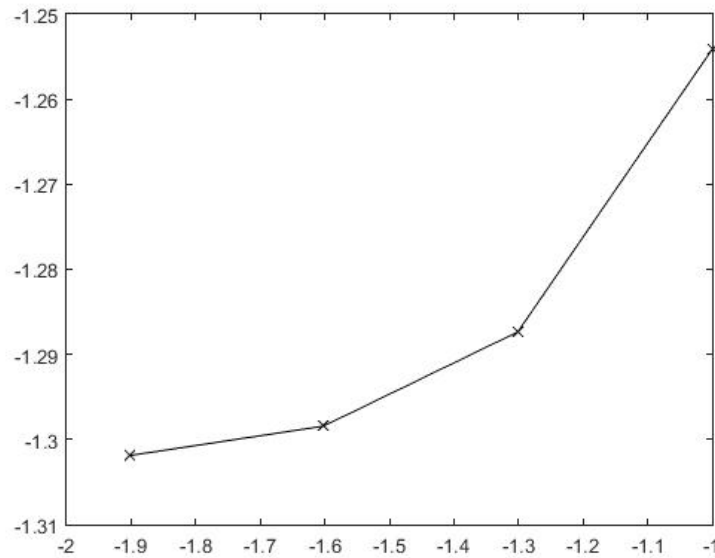
$$\text{err}(t = 10) = [0.0067 \ 0.0020 \ 5.6787e - 004 \ 1.6004e - 004]$$

$$\text{err}(t = 50) = [0.0557 \ 0.0516 \ 0.0503 \ 0.0499]$$



Here we can see the error vector after refining four times the initial mesh and using the elliptic problem to solve the equation.

$$errElliptic = [ 0.0557 \ 0.0516 \ 0.0503 \ 0.0499 ]$$



As we can see we get the same values of the error when we solve the problem using a parabolic problem or an elliptic problem.