Computational Mechanics Tools
Assignment 2 - Nonlinearity

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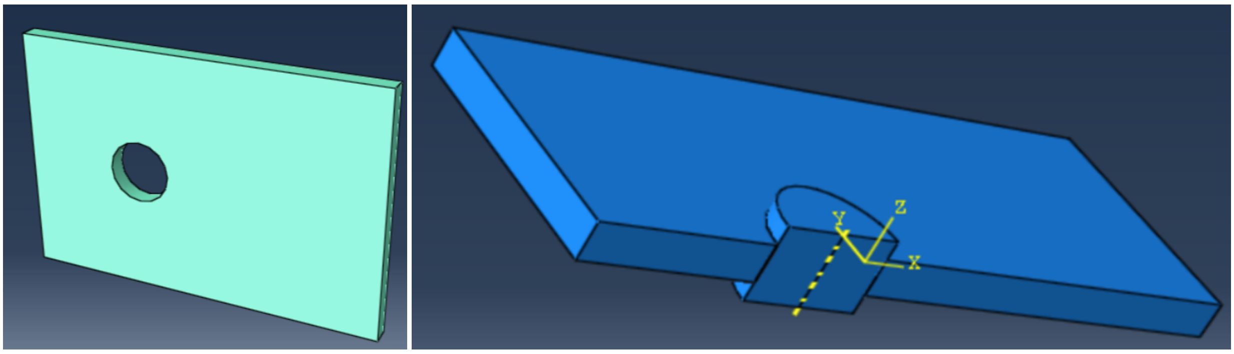
Abstract

Two tutorial using *ABAQUS* software were given to perform a calculation of the stresses on a steel plate with a hole submitted to axial tensile force in one case, and in the later, a fixed pin is introduced in the hole and the friction between them is taken into account. For both exercises the materials used were modelled using three different laws, namely *elastic*, *elastic-perfectly plastic* and *elasto-plastic with hardening*. It was checked in the models that for both *elastic-perfectly plastic* and *elasto-plastic with hardening* the registered forces and stresses are lower, being higher for the model with hardening behaviour than perfectly plastic one as expected.

Introduction

The software *ABAQUS* has different environments suitable to different problems such as Dynamics (varying implicit or explicit integration), Fluid Mechanics, Electromagnetism and some the typical FEM versions. In our case, we use the *ABAQUS-CAE* "Complete Abaqus Environment". It is a software application used for both the modelling and analysis of mechanical components and assemblies (pre-processing) and visualizing the finite element analysis result. The problems to be solved were the stresses of a plate with a hole subjected to axial tensile force as shown in Figure 1a and a case with a fixed pin considering the friction between the pin and the plate as shown in Figure 1b (in this case only half of the problem was modelled due to its symmetry).
The conditions in each of the models are as follows:

- Plate with hole
  - both lateral planes of the plate (of width equal to the thickness of the plate) have an imposed displacement with opposite signs and same directions (generating a tension on the plate) of 0.05 mm magnitude;
  - on the same lateral planes, at the mid-height, this mid-line has been fixed in both Y and Z direction (vertical direction and normal to the plate, respectively). A point on this line is selected to register both displacements and reactions.
- Plate with pin
  - the right lateral plane of the plate (of width equal to the thickness of the plate) have an imposed displacement of 0.1 mm;
  - the axis of revolution to generate the pin is fixed in X direction;
  - the lower plane of the plate and the pin are fixed in both Y and Z direction (vertical direction and normal to the plate, respectively). A point on the edge of the plate inside, also in this plane is selected to register both displacements and reactions.



(a) Plate with hole (b) Plate with pin

Figure 1: Problems to be solved.

For each of this two cases the materials of the plate and the fixed pin were described using *elastic*, *elastic-perfectly plastic* and *elasto-plastic with hardening*. These material descriptions are know as *constitutive laws* and relate *stress with deformation*. This models are presented in Figure 2 first case (*elastic*, red line in the graph) relates directly the deformation times a constant to the stress, the second one (*elastic-perfectly plastic*, red+green line) has an elastic description until an upper boundary known as the yield stress (σ_y) and the last (*elasto-plastic with hardening*, red+blue line) has an elastic description until a yield stress boundary that modifies the slope between σ and ε but continue to increase the stress with increasing deformations.
So a general comment on these three descriptions will be that for a given deformation bigger than the yield stress ($\varepsilon > \varepsilon_y$), the stresses on the different model would be from higher to lower registered in (1) *elastic*, (2) *elasto-plastic with hardening* and (2) *elastic perfectly plastic*.

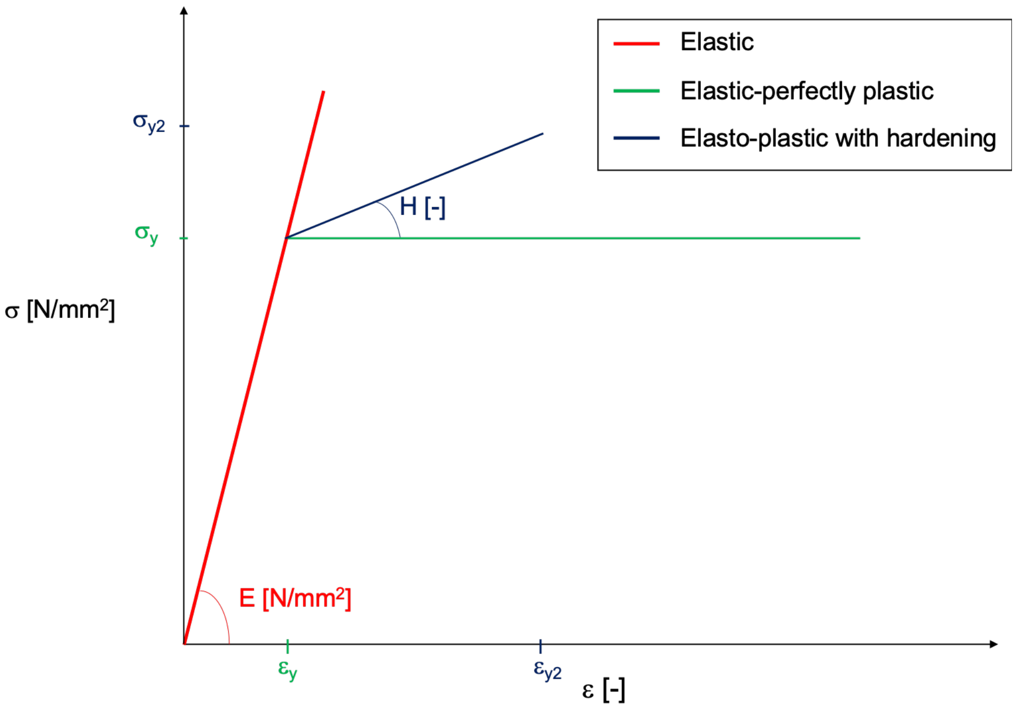


Figure 2: Description of the materials strain-stress behaviour.

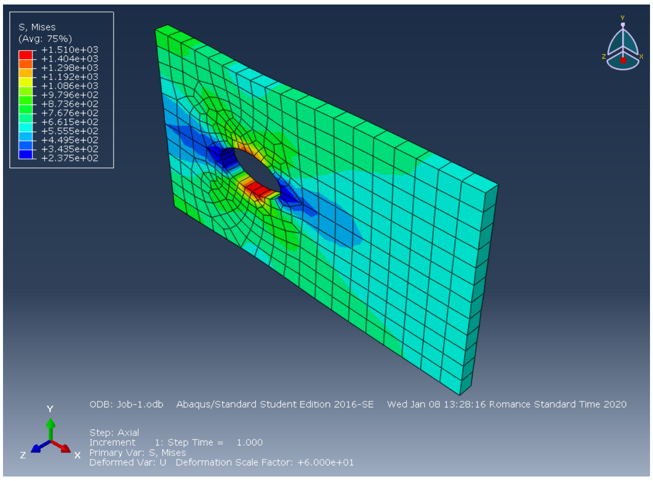
Plate with hole

The plate with a hole from Figure 1a is solved using different constitutive laws as described before, and in Table 1 their main characteristics are described. In Figure 3 the von Mises stress distribution on the plate for the different cases are shown, and in Figure 4 the force-displacement relationship for a node with both applied displacement and reaction force is shown. All the figures are plotted in a bigger scale at the end of the document.

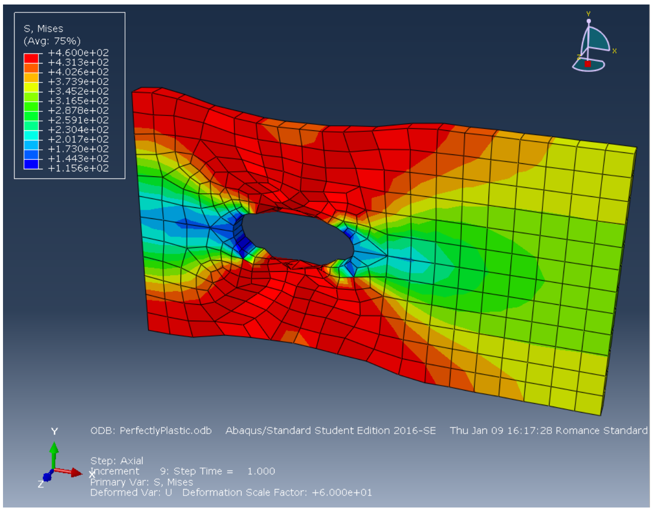
From Figure 3 it is seen that the stress distribution is similar in all cases, registering an increase of stresses nearby the hole forming a distribution similar to an X with center in the hole. This is repeated in every case even though the variations of the constitutive laws, and the color maps represent when this strains reach any yield strain. The biggest maximum stress is registered in the *elastic* model with a maximum of approximately $1500 N/mm^2$ and the lowest maximum stress is registered in the *elastic-perfectly plastic* model with a maximum of $460 N/mm^2$ as expected.
From Figure 4 it can be seen as predicted in the explanation of the constitutive laws that the *elastic* material would register the biggest stresses and therefore the biggest forces (approximately $800N$ as well, and then, the *elasto-plastic with hardening* would have the second bigger stresses and finally the *elastic-perfectly plastic* would register the lowest stresses and also forces. Then the figure register the expected results.

Case -	Constitutive law -	Elastic Modulus E [N/mm^2]	Shield Stress σ_y [N/mm^2]	Total Strain ε_y [-]	Shield Stress (2) σ_{y2} [N/mm^2]	Total Strain (2) ε_y [-]
1	Elastic	$2.1 \cdot 10^5$	-	-	-	-
2	Perfectly plastic	$2.1 \cdot 10^5$	460	0.00219	-	-
3	Hardening 1	$2.1 \cdot 10^5$	460	0.00219	520	0.00719
4	Hardening 2	$2.1 \cdot 10^5$	460	0.00219	520	0.00419

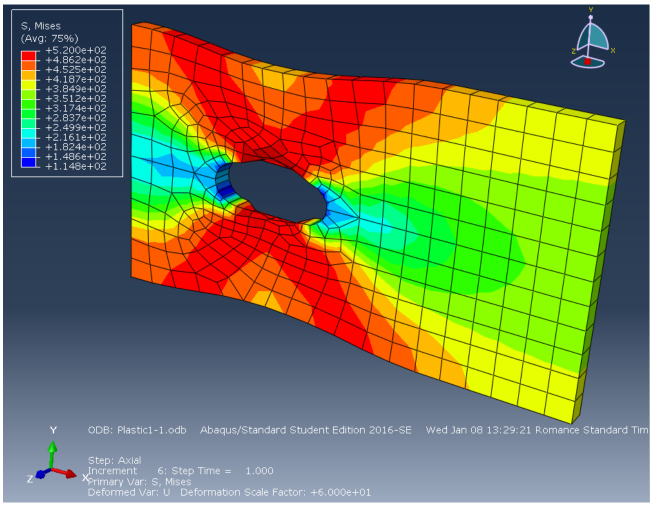
Table 1: Constitutive laws of the materials modelled.



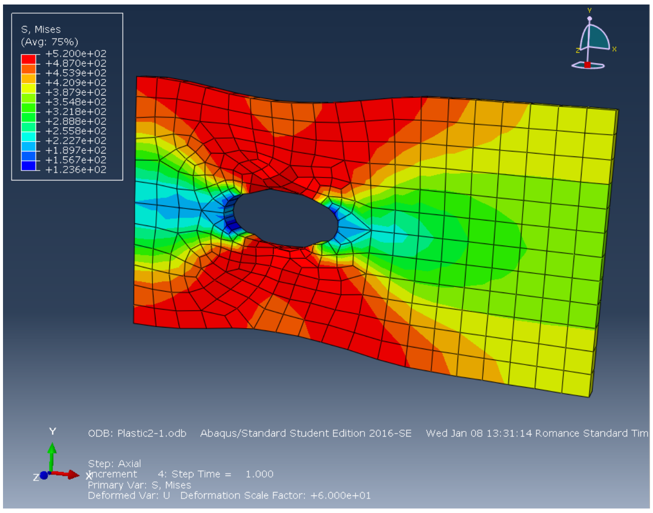
(a) Case 1: Elastic.



(b) Case 2: Elastic-perfectly plastic

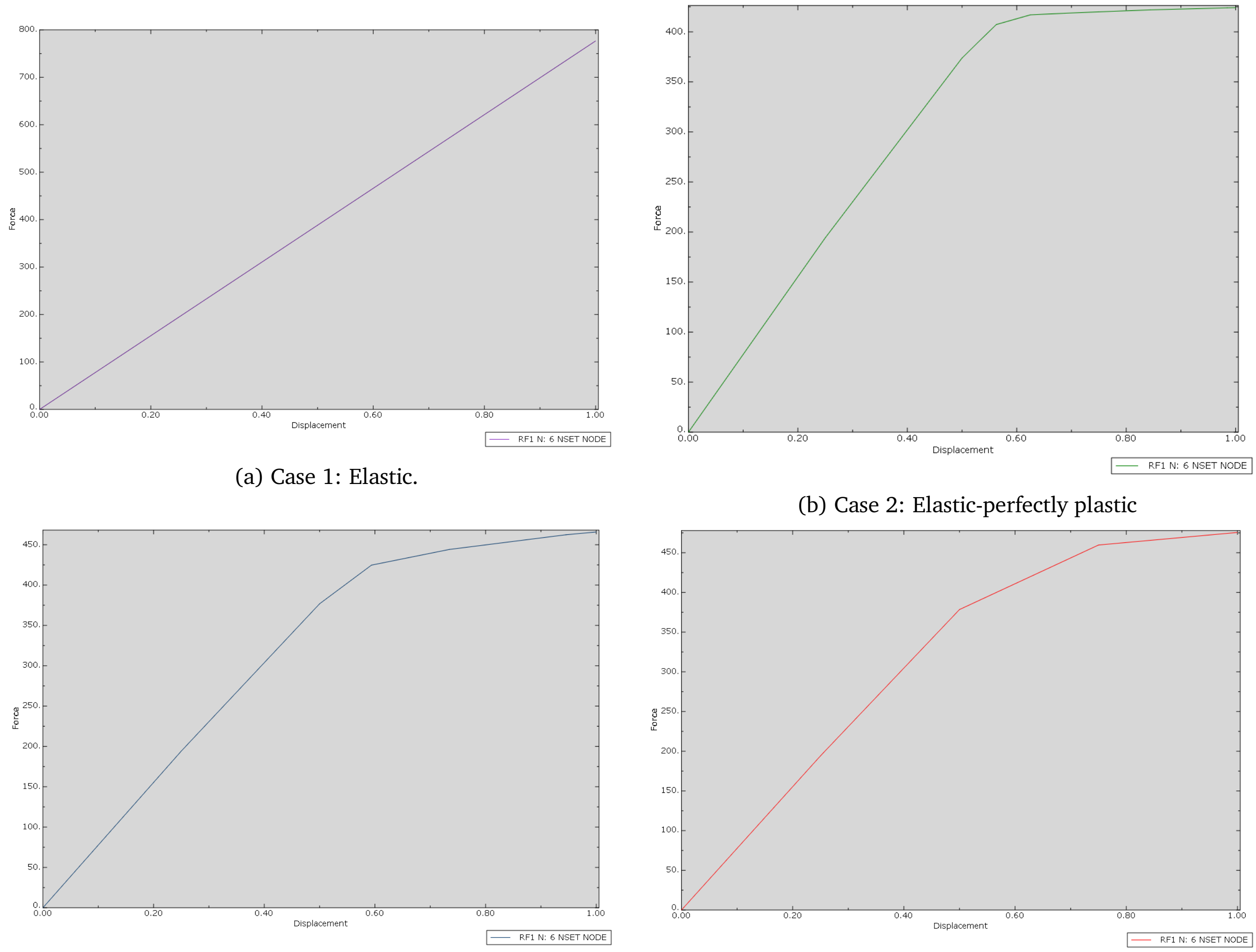


(c) Case 3: Hardening 1.



(d) Case 4: Hardening 2.

Figure 3: von Mises stress distribution on the plate for different constitutive laws.



(a) Case 1: Elastic.

(b) Case 2: Elastic-perfectly plastic

(c) Case 3: Hardening 1.

(d) Case 4: Hardening 2.

Figure 4: Force-displacement relationship for a point in the boundary.

Plate with fixed pin

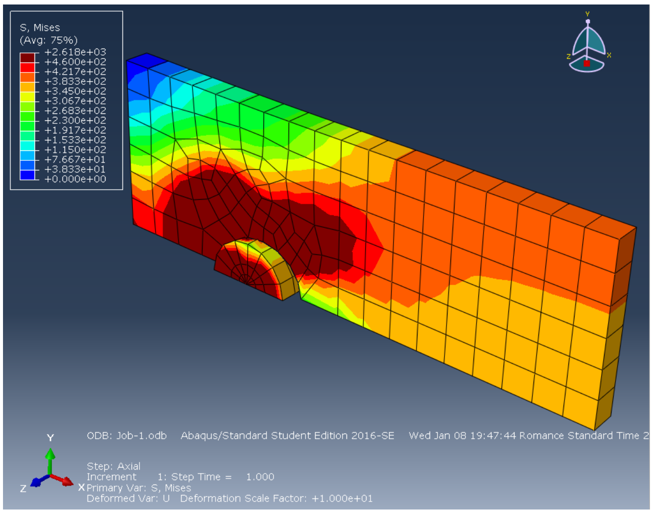
The second case of analysis presented in Figure 1b, the plate with a fixed pin, is solved using different constitutive laws and combinations of them between the plate and the fixed pin. In Table 2 a summary of their properties is presented. In Figure 5 the von Mises stress distribution on the plate for the different cases are shown, and in Figure 6 the force-displacement relationship for a node with both applied displacement and reaction force is shown. In all of the cases the exaggeration factor is setted equal to ten. All the figures are plotted in a bigger scale at the end of the document.
From Figure 5 it is seen that in all the three figures, considering that the tensile deformation of the plate (in the right side of the figure) makes the contact between the plate and the pin (in the opposite side, left) to register higher stresses. In the first case, when the pin is elastic, it registers stresses approximately up to $2600\ N/mm^2$ generating a big surface of the plate to be over its yield stress limit ($460\ N/mm^2$). Afterwards, when the pin is described by hardening functions, it reaches of course the yield stress and the stresses in the plate also reaches its yield stress but the surface reaching it is considerably lower.
In the case of the force-deformation graphs, it should be considered that the plate contains the node of interest, therefore the tensions on the plate would be the ones generating the force. From Figure 6 it can be seen that the *elastic pin* model (Case 1) registered the biggest force in the plate, which is closely related to what was mentioned on the previous paragraph.

Case	Part	Constitutive law	Elastic Modulus	Shield Stress	Total Strain	Shield Stress (2)	Total Strain (2)
-	-	-	E [N/mm^2]	σ_y [N/mm^2]	ε_y [-]	σ_{y2} [N/mm^2]	ε_y [-]
1	Plate	Perfectly plastic	$2.1 \cdot 10^5$	460	0.00219	-	-
	Pin	Elastic		-	-	-	-
2	Plate	Hardening 1	$2.1 \cdot 10^5$	460	0.00219	520	0.00719
	Pin	Hardening 1		900	0.00428	1000	0.00628
3	Plate	Hardening 1	$2.1 \cdot 10^5$	460	0.00219	520	0.00719
	Pin	Hardening 2		320	0.00152	400	0.00652

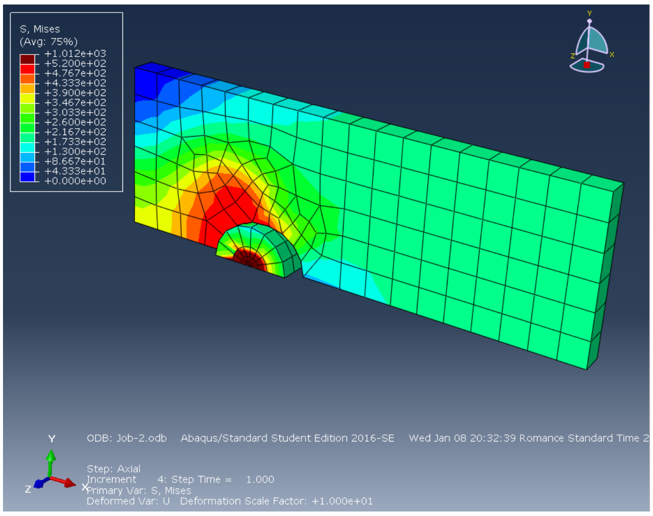
Table 2: Constitutive laws of the materials modelled.

Conclusions

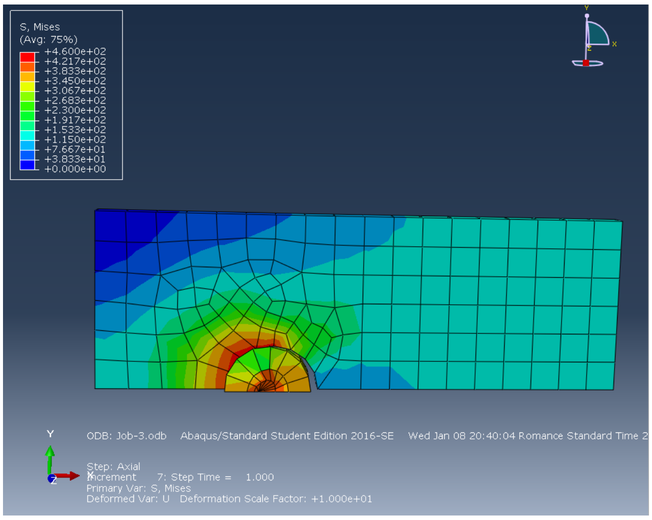
The assignment consisted in solving two models of a plate subjected to tensile stresses. In the first a deformation was imposed from the boundaries, half of the displacement for each side generating a tensile force in the plate. Four different material descriptions and constitutive laws were used registering as expected highest forces when using elastic description and reducing them when plastic descriptions were taken into account. The second model considered the intrusion of a fixed pin in the hole, and used the friction force between the plate and the pin to describe the interaction among them. In this case the displacement was imposed only in one side of the plate registering bigger forces on the opposite side of this application due to the presence of the fixed pin. Similar results considering both forces and stresses were registered in both cases for similar material description conditions, as highest forces appeared on the elastic description of the pin.



(a) Case 1: Plate: Perfectly plastic + Pin: Elastic.

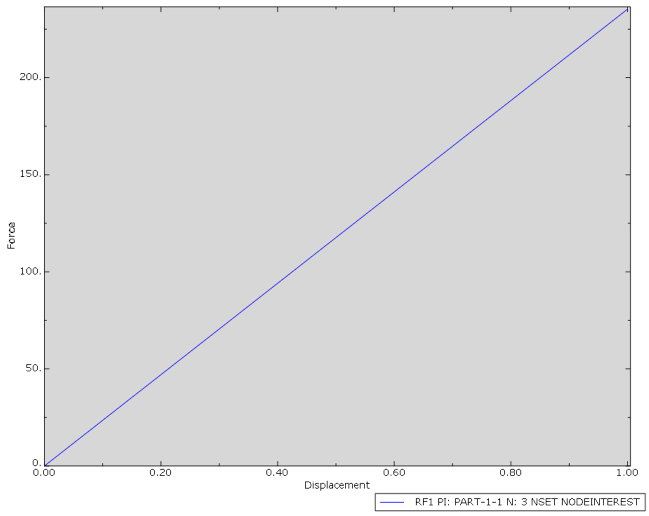


(b) Case 2: Both hardening 1.

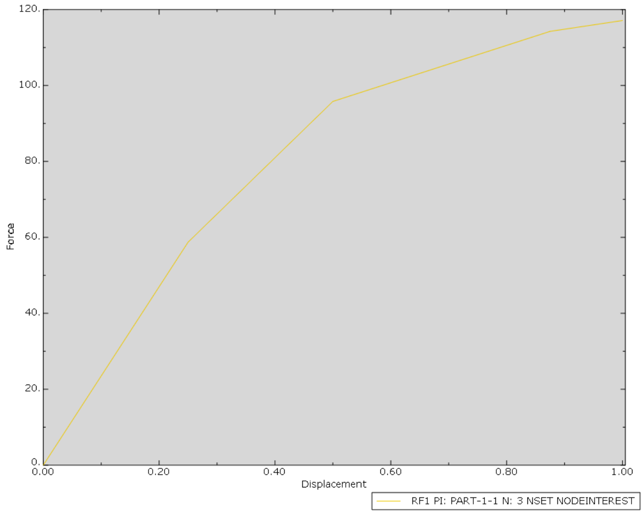


(c) Case 3: Plate: Hardening 1 + Pin: Hardening 2.

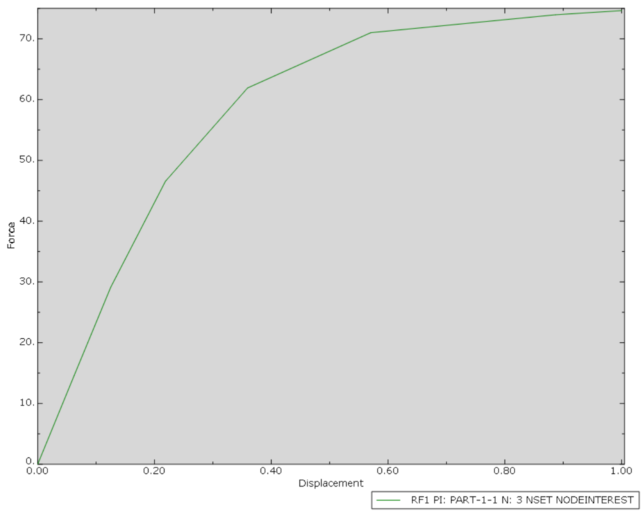
Figure 5: von Mises stress distribution on the plate for different constitutive laws.



(a) Case 1: Plate: Perfectly plastic + Pin: Elastic.



(b) Case 2: Both hardening 1.



(c) Case 3: Plate: Hardening 1 + Pin: Hardening 2.

Figure 6: Force-displacement relationship for a point in the boundary.

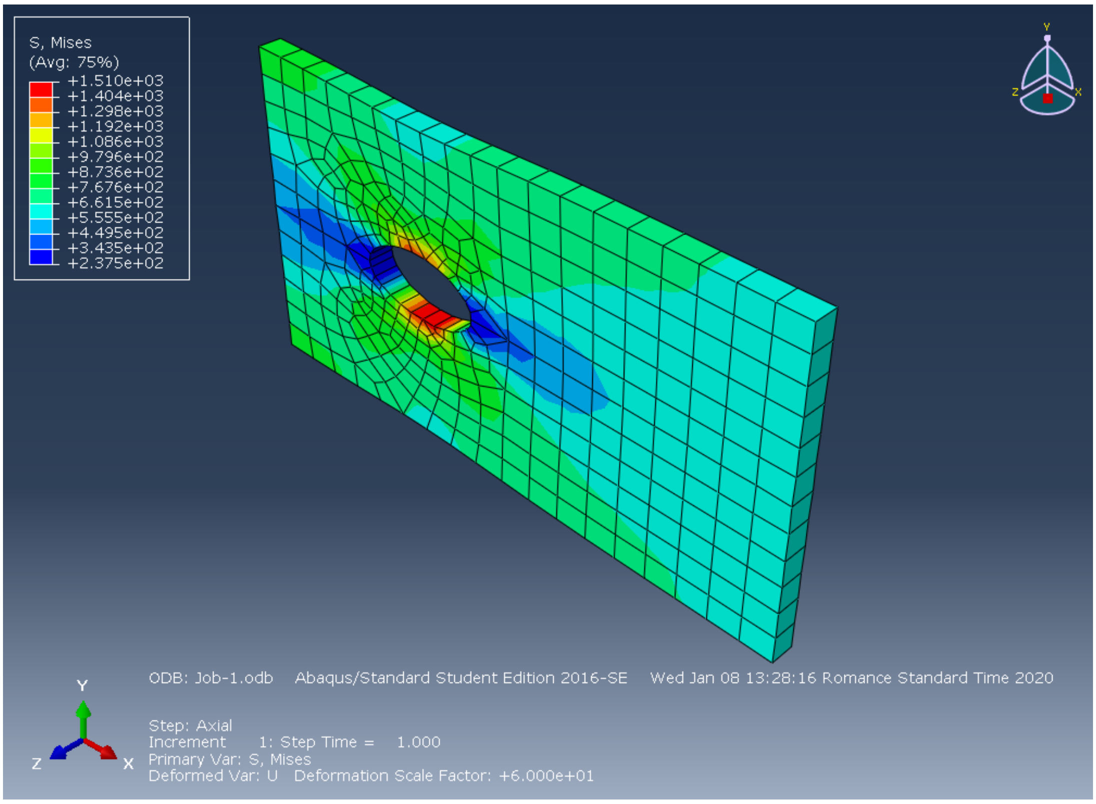


Figure 7: Exercise 1: Plate with hole. Case 1. Elastic.

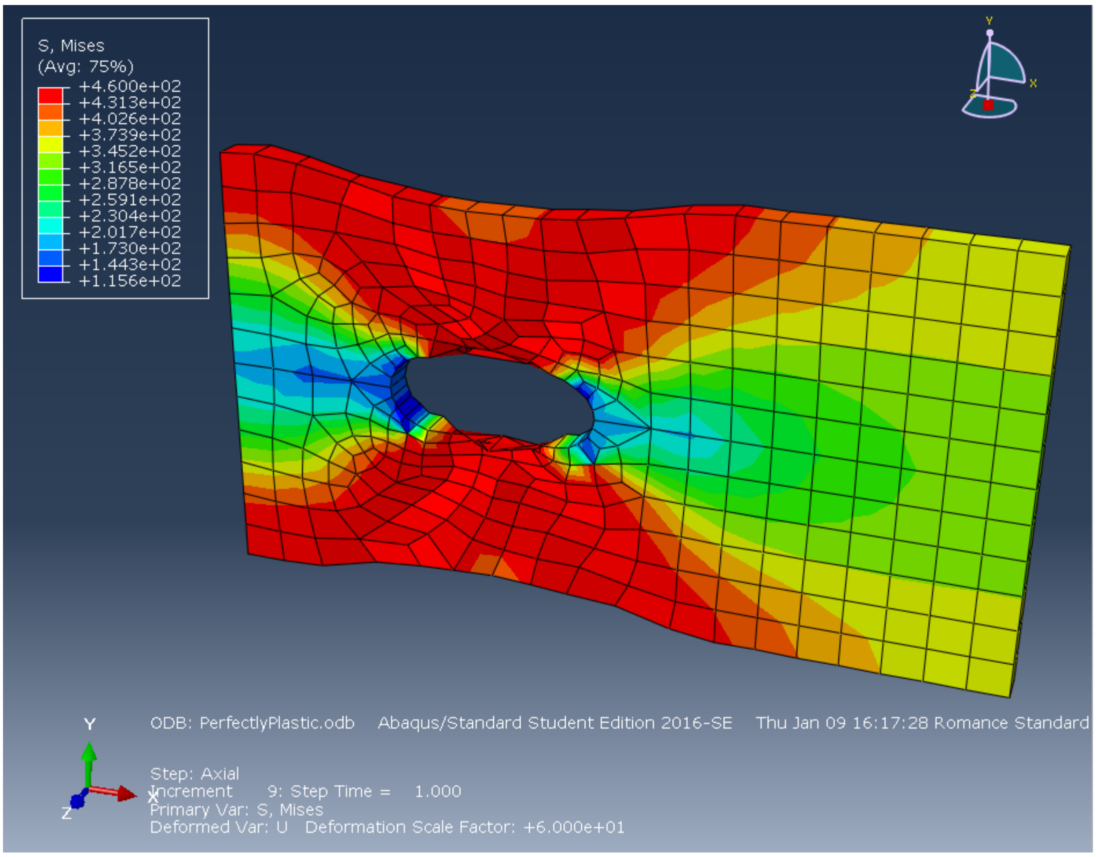


Figure 8: Exercise 1: Plate with hole. Case 2. Elastic-perfectly plastic.

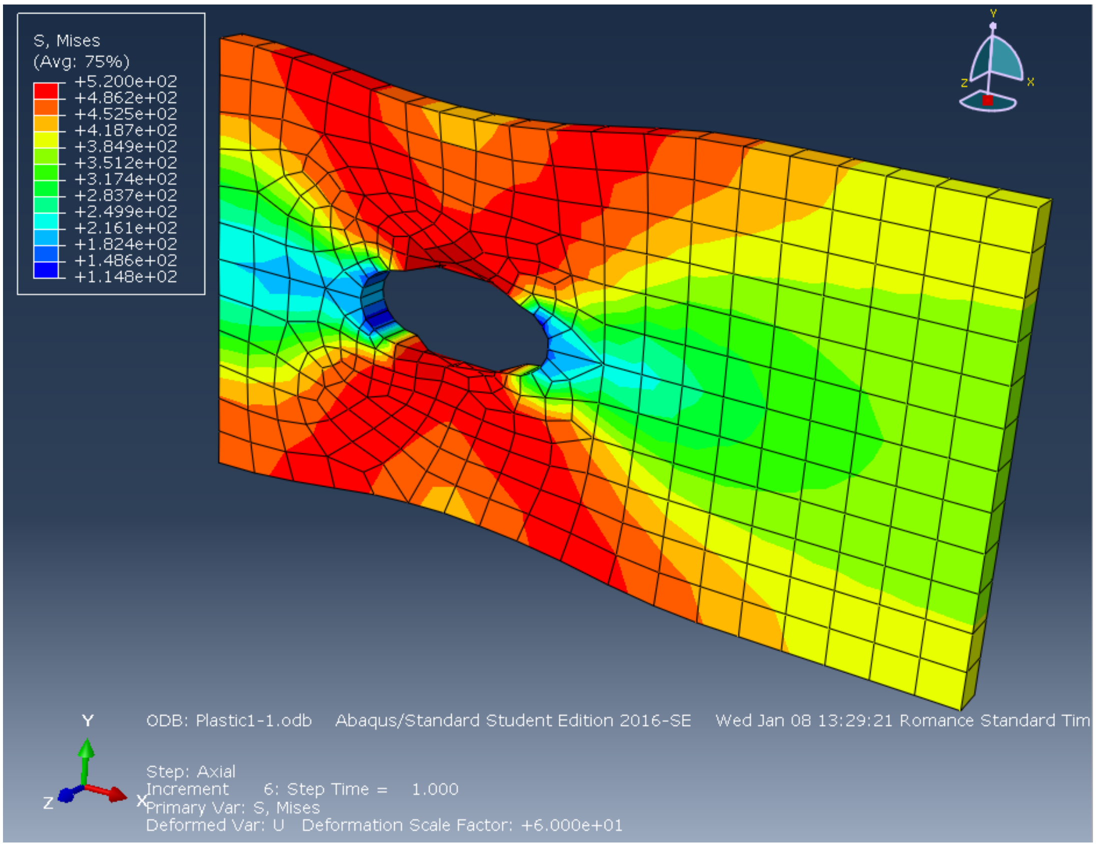


Figure 9: Exercise 1: Plate with hole. Case 3. Hardening 1.

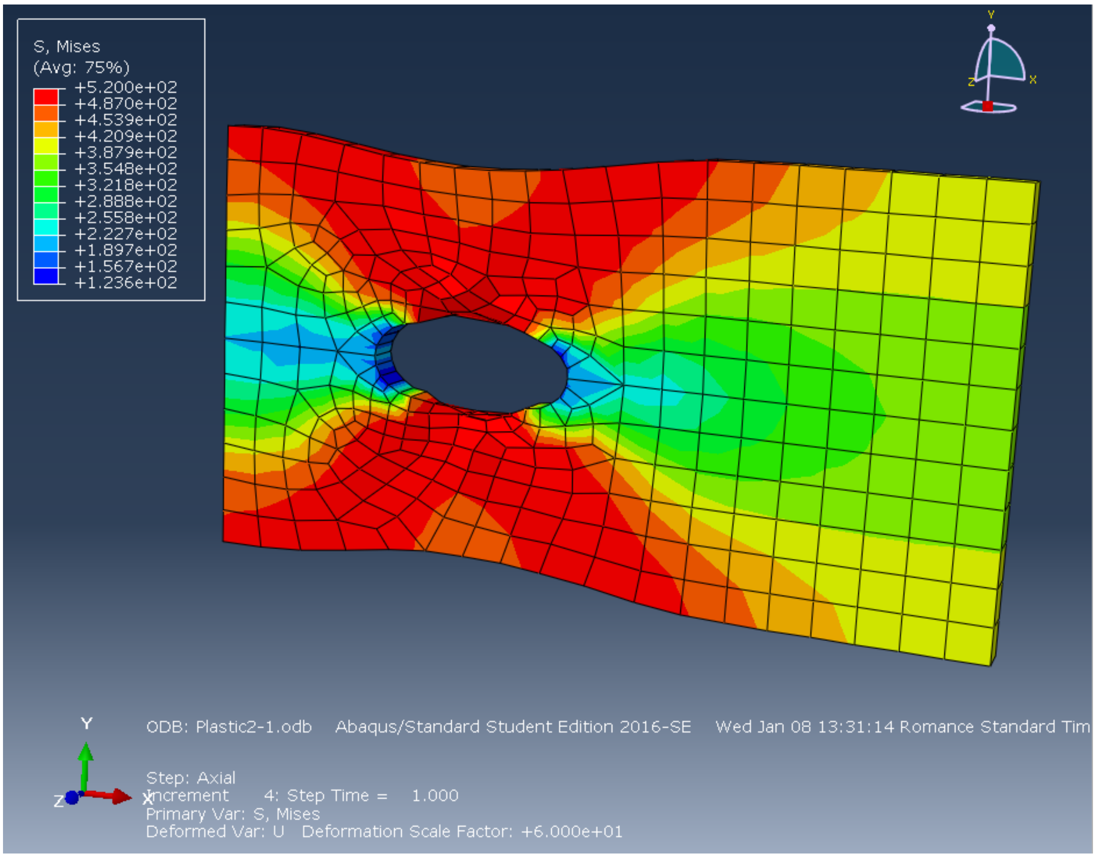


Figure 10: Exercise 1: Plate with hole. Case 4. Hardening 2.

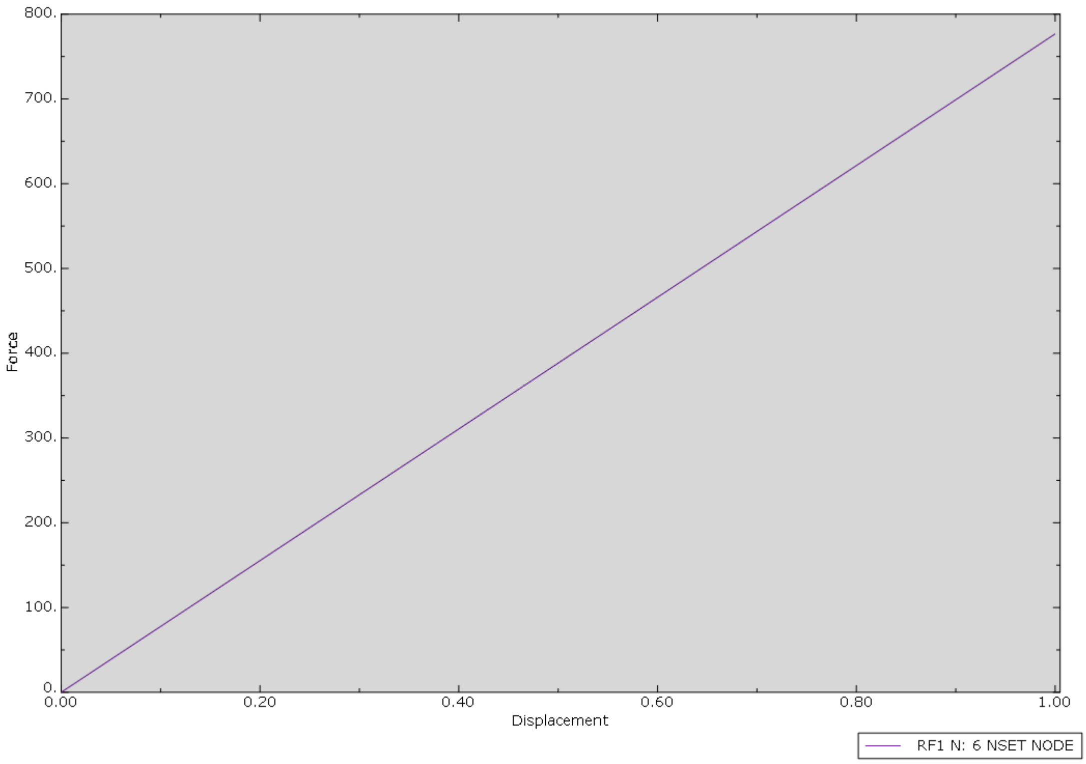


Figure 11: Exercise 1: Plate with hole. Case 1. Elastic. Displacement

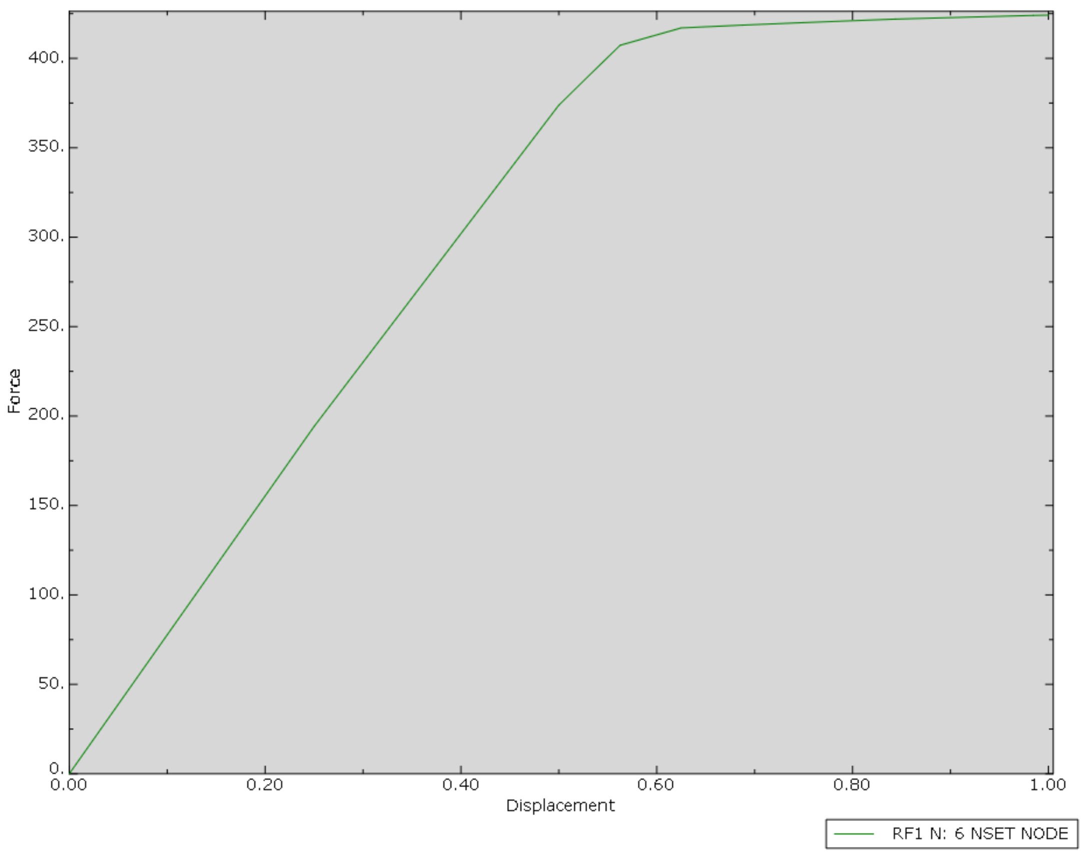


Figure 12: Exercise 1: Plate with hole. Case 2. Elastic-perfectly plastic. Displacement.

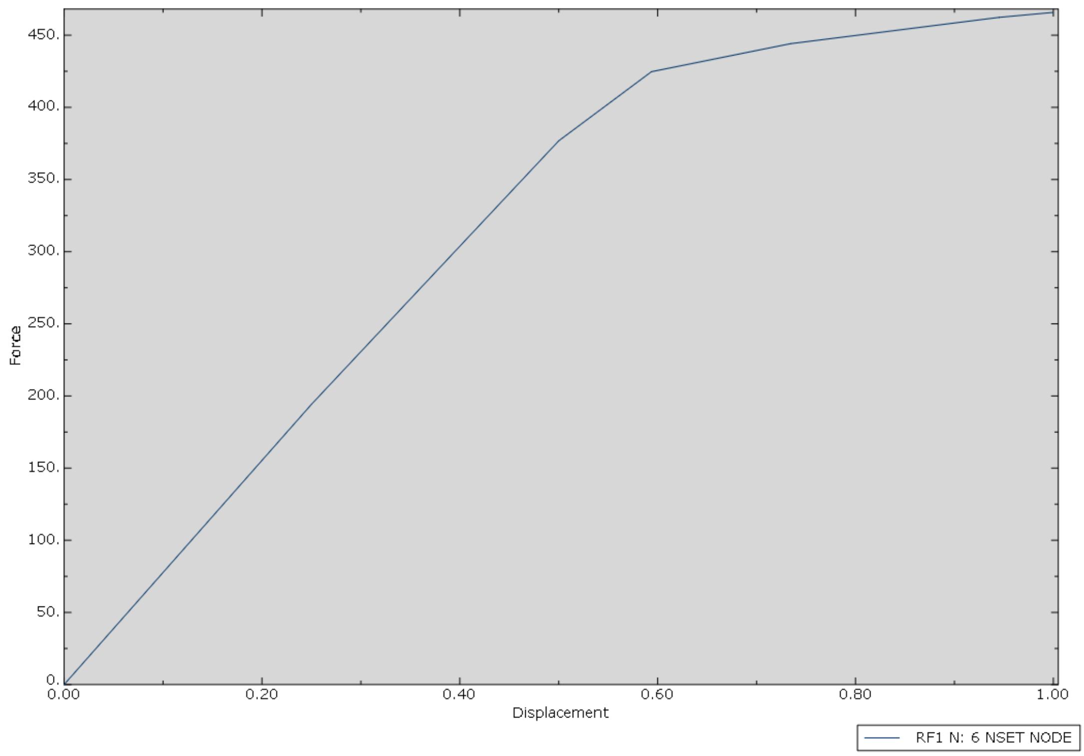


Figure 13: Exercise 1: Plate with hole. Case 3. Hardening 1. Displacement.

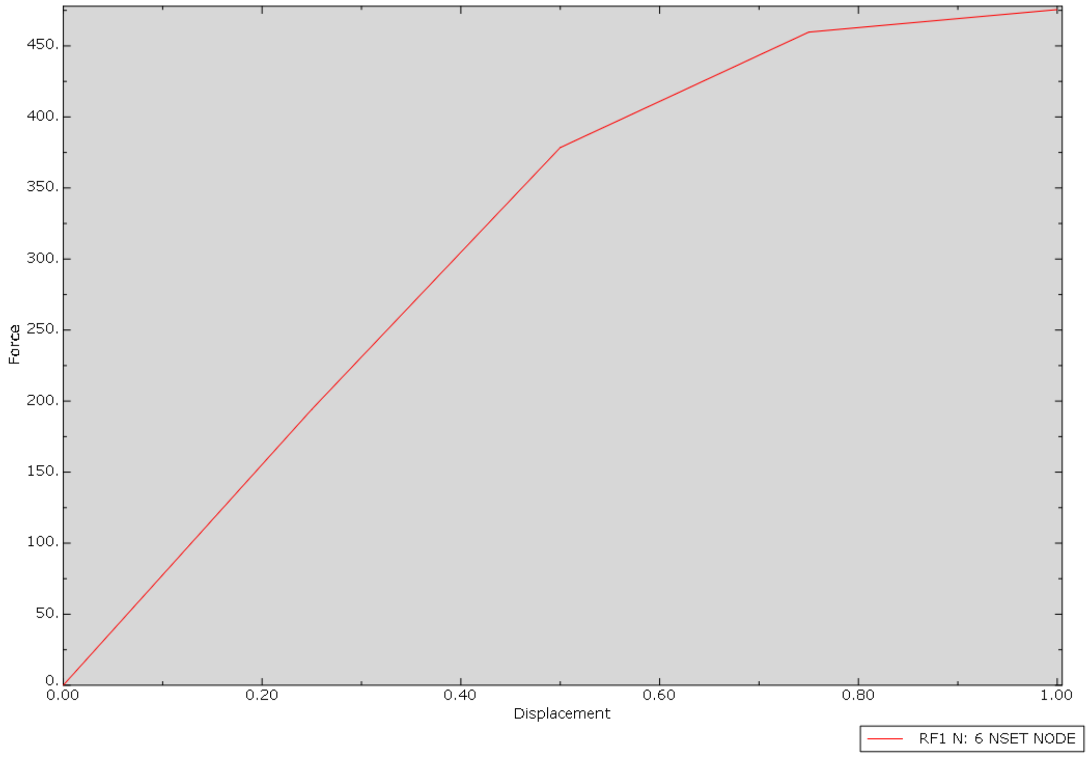


Figure 14: Exercise 1: Plate with hole. Case 4. Hardening 2. Displacement.

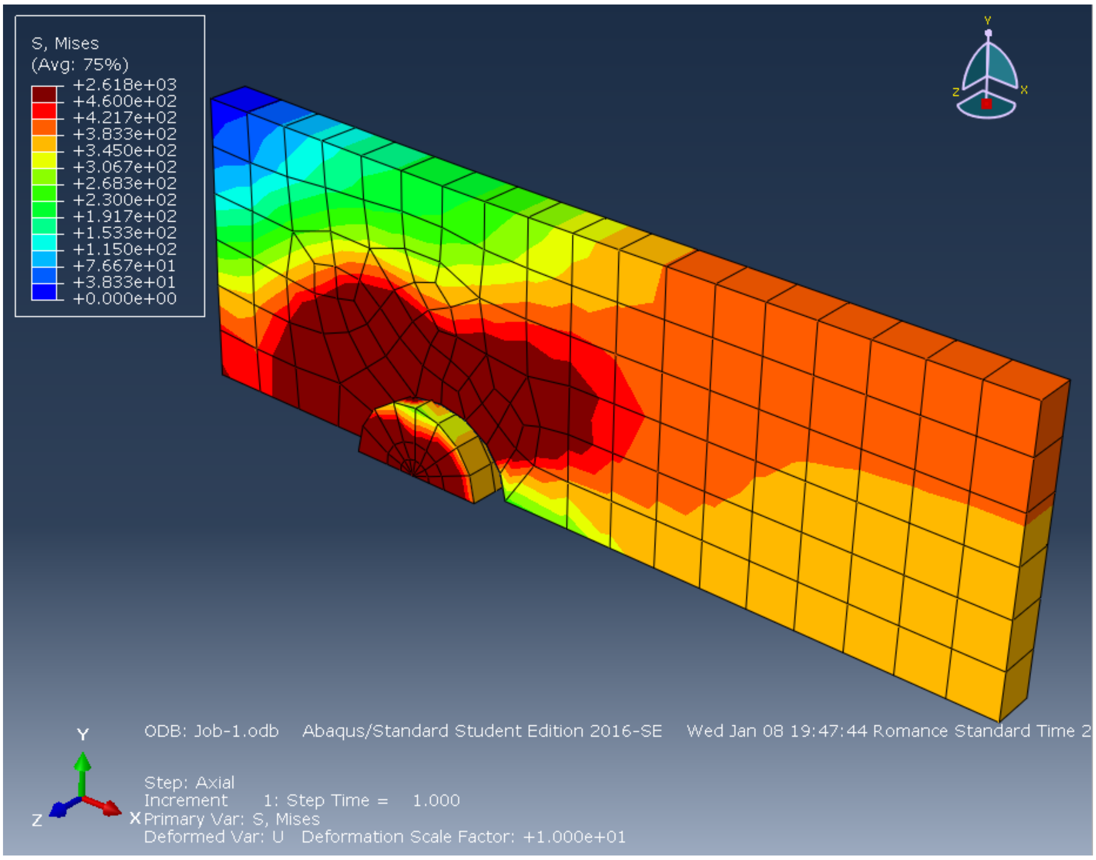


Figure 15: Exercise 2: Plate with pin. Case 1: Plate: Perfectly plastic + Pin: Elastic. Stresses.

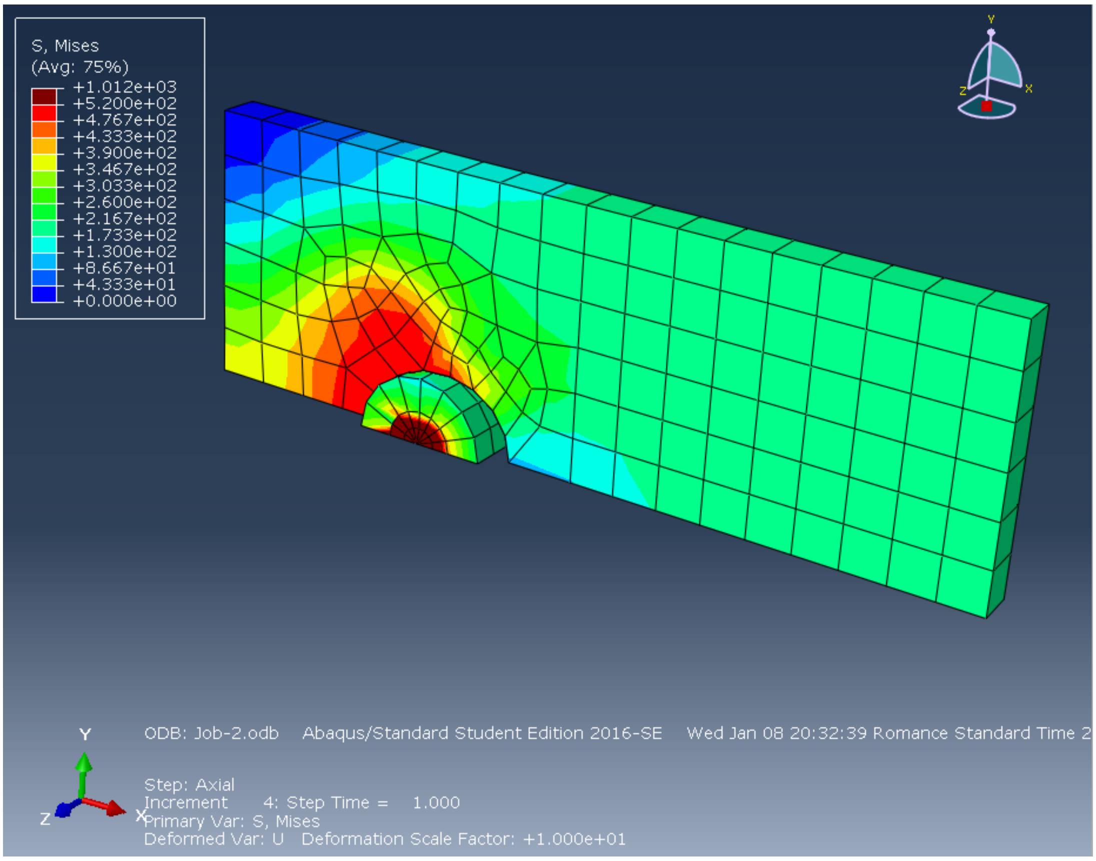


Figure 16: Exercise 2: Plate with pin. Case 2: Both hardening 1. Stresses.

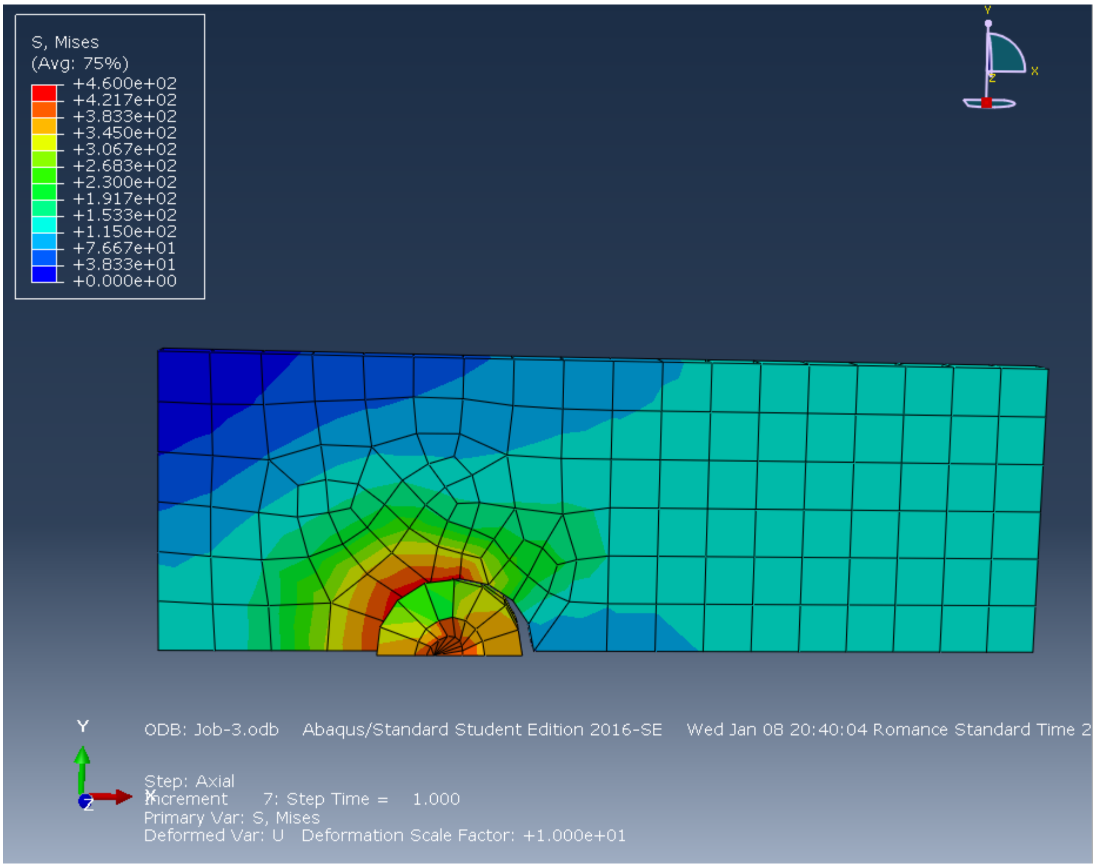


Figure 17: Exercise 1: Plate with pin. Case 3: Plate: Hardening 1 + Pin: Hardening 2. Stresses.

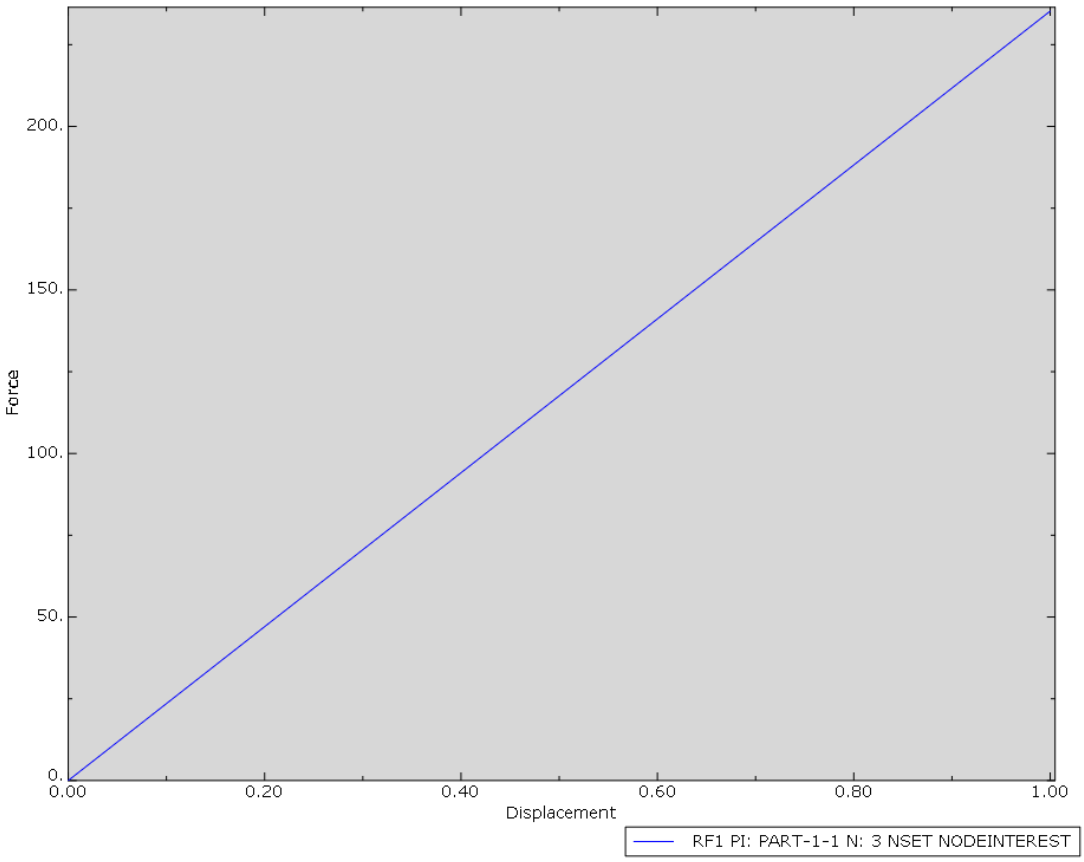


Figure 18: Exercise 2: Plate with pin. Case 1: Plate: Perfectly plastic + Pin: Elastic. Deformations.

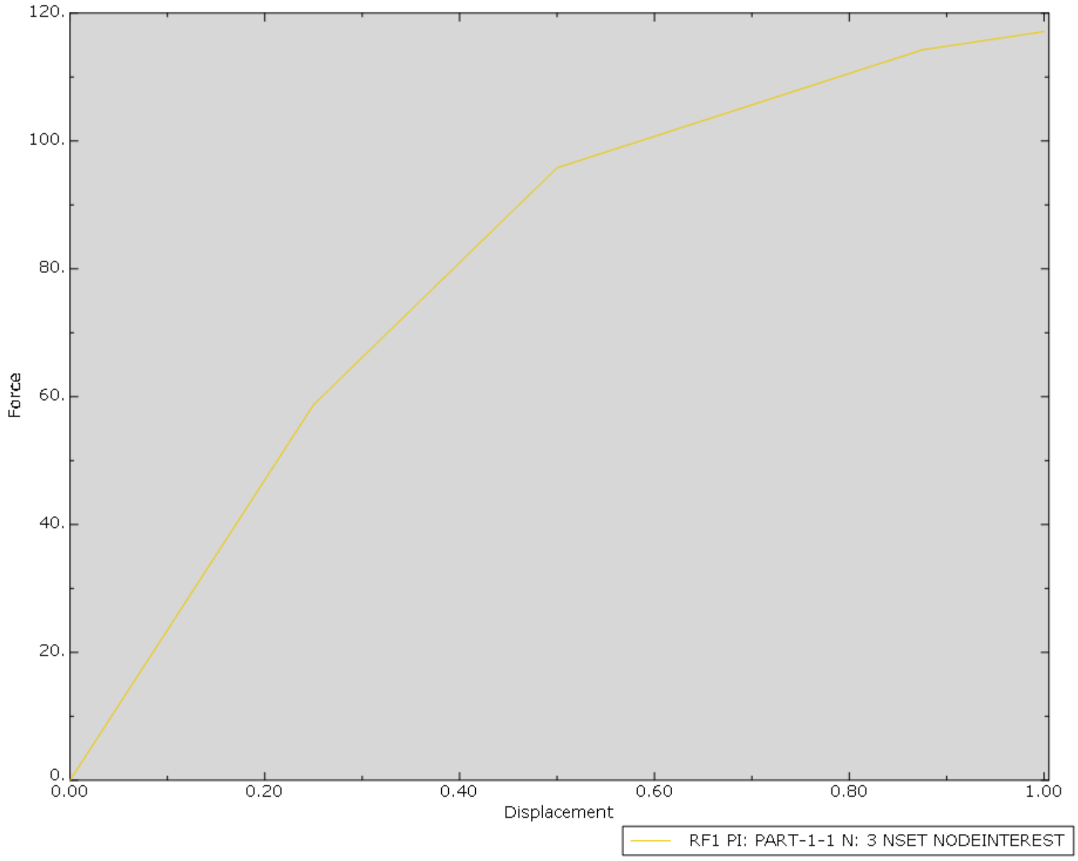


Figure 19: Exercise 2: Plate with pin. Case 2: Both hardening 1. Deformations.

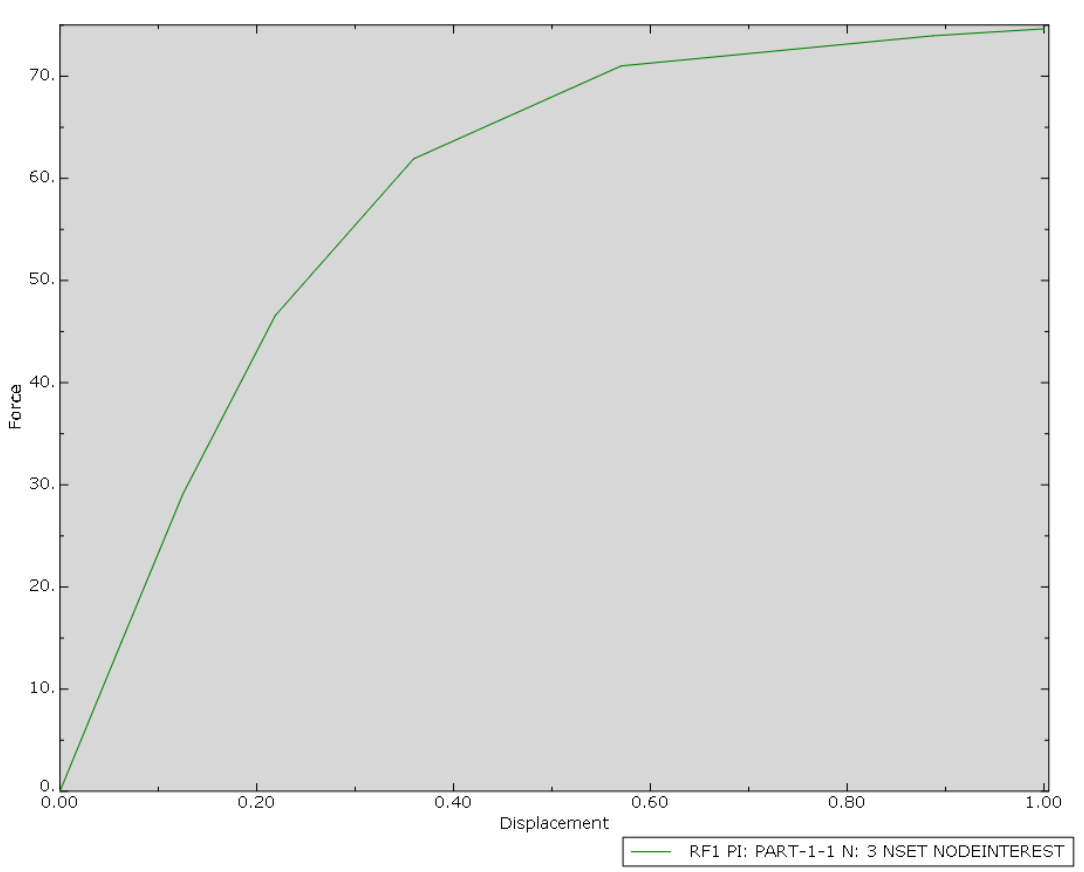


Figure 20: Exercise 2: Plate with pin. Case 3: Plate: Hardening 1 + Pin: Hardening 2. Deformations.