COMPUTATIONAL MECHANICS TOOLS
Master of Science in Computational Mechanics/ Numerical Methods
Fall Semester 2015
Assignment 3: Non-linear Elastic Block
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Task 1· Identify in the code (files, lines) the following items:

a) The definition of the example (loading, geometry).

File: preprocessing.m
- Loading and geometry lines 8 to 59.
- Material properties lines 86 to 101.

b) The choice of solution method (Newton's method with or without line-search.

File: main_incremental_iterative.m
- Method line 20 (flag options.method=1 for Newton's method).
- Line seach selection line 21 (flag options.linesearch).

File: main_buckling.m
- Method line 18 (flag options.method=1 for Newton's method).
- Line search selection line 20 (flag options.linesearch).

c) The implementation of the solution method.

File: Equilibrate.m
- Lines 9 to 94 (particularly for newton's method; case 1 lines 36 to 72).

d) The implementation of the inc.remental-iterative strategy, with smart initial guesses for imposed displacements

File: main_incremental_iterative.m
- Lines 44 to 48.

e) The introduction of random perturbations in the initial guesses of the solution method

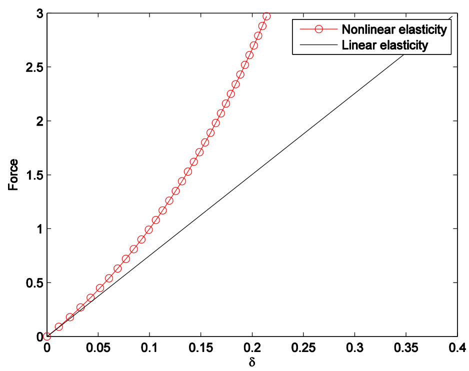
File: main_incremental_iterative.m
- Line 50 x=x+rand(size(x))*.001

File: main_buckling.m
- Line 76 x=x+rand(size(x))*.001
- Lines 53 to 56

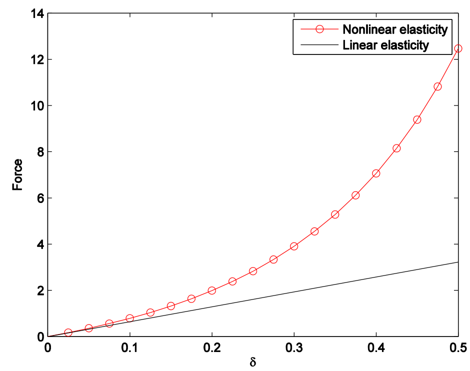
Task 2· Report and discuss a selection of results that illustrate the features of the nonlinear model vis a vis the following feature of linear models:

- Proportionality of the response with respect to the loads

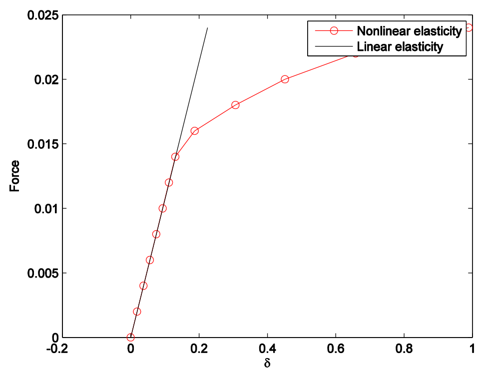
In Figure 1 we can see that while for linear elasticity force and displacement follow a linear relation with a slope related with the Young modulus ($E = \sigma/\varepsilon$) , the nonlinear analysis does not present this property. In this same figure we can see the behavior of two different nonlinear materials (Neohookean in a and b; transversely isotropic in c and d) being that for the first one given the same force the Neohookean material has a lesser response in displacement, while for the transversely happens the opposite. It is also notable that for small loads the nonlinear materials have a behavior similar to a linear one



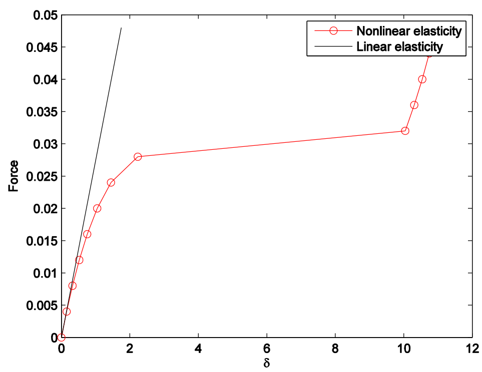
(a) Example 0



(b) Example 1



5(c) Example 4

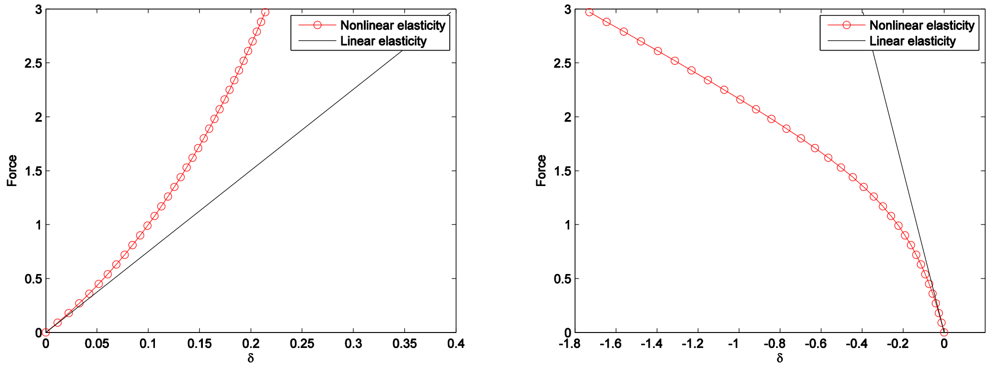


(d) Example 5

Figure 1. Deformation vs force diagrams for incremental-iterative method with line search and random perturbations

- Symmetry of the response with respect to the sign of the loads

In figure 2 we can see that the linear model keeps symmetry of the displacement in front of the sign of the force, while the nonlinear does not. Comparing both models we can also see that for a given force with positive sign the displacement is smaller in the case of nonlinear material, while for that same given force with negative sign happens the opposite



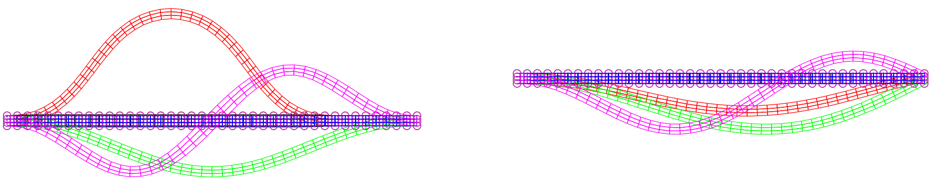
(c) Example 0 with positive force (d) Example 0 with negative force

Figure 2. Deformation vs force diagrams for incremental-iterative method wioth line search and random perturbations

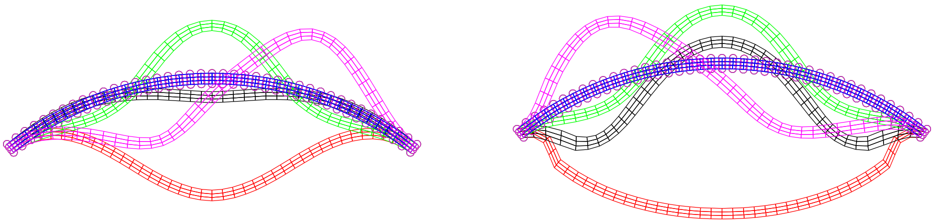
- Uniqueness of solutions

While linear elasticity only accounts for one possible solution, when using nonlinear models it appears multiple solutions, usually related to buckling effects. Buckling is a phenomena in where a single load case can be hold by different states of equilibrium. This can be seen in figure 3 as for every example we have 3 different nonlinear solutions.

It is also important to notice that usually each buckling mode has a symmetric mode.



(a) Example 2 (b) Example 3

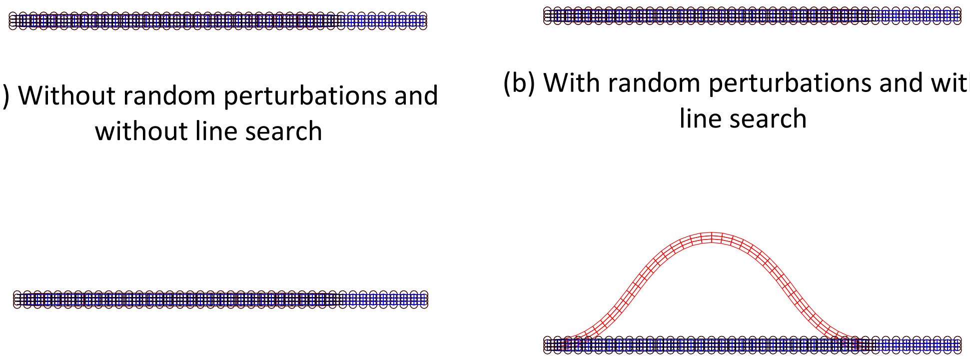


(c) Example 4 (d) Example 5

Figure 3. Solution of the deformations with buckling modes random perturbation and line search

- Stability of the solutions

Related with the fact explained in the previous point [multiple solutions] we can have that some solutions are not stable, meaning that if a perturbation is applied the equilibrium changes and does not recover the original state.



(a) Without random perturbations and without line search

(b) With random perturbations and with line search

(c) Without random perturbations and with line search

(d) With random perturbations and line search

Figure 4. Solution of the deformations for example 2

In figure 4 we can see that for a case with symmetric geometry loading and BC's, it may be necessary to compute the solution using random perturbation with line search in order to eliminate unstable equilibrium (a, b, and c) and get a stable equilibrium (d). It is important to remark that due to the random nature of the perturbations we may also obtain the symmetric solution. However linear elasticity cannot differ between stable and unstable equilibriums.

- Conclusions

Linear elasticity is built on the assumption of small displacements and therefore the nonlinear terms can be neglected as their contribution to the solution is much smaller than linear terms. The validity of this assumption is easily checkable in the force-displacement diagrams, as for small displacement both models have the same behavior where for larger ones they differ.

While linear elasticity is easier to solve, nonlinear models present some extra difficulties that make the use of them needy of more awareness, as could be the existence of multiple solutions for one single case or obtaining unstable solutions. For this last point it is interesting to introduce line-search methods or small mesh perturbations in order to get rid of the unstable equilibriums and be left only with the stable ones.

Concluding, although nonlinear elasticity is always valid and holds for any case, the use of linear elasticity should be preferred if small displacements assumption can be made.