Computational Mechanics Tools: Assignment 3

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8th January 2016

This assignment focuses on non-linear elasticity and his application in some cases where linear elasticity is not a good approximation. The code to solve it has been delivered so in this assignment will be only discussed the results for different examples. It is used the incremental iterative code to obtain the stress-strain curve using linear and non-linear elasticity to compare them.

0. In this example, is treated a block where there is applied two cases of force: first is compressed and later is stretched changing only the direction of the force. In this case has been used the Newton-Raphson method without line-search because is not necessary as it is a simple case where there are not abrupt changes in the stress-strain curve. The two deformations obtained are the following:

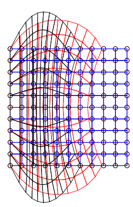

Figure 1: Stretch deformation for force 3e0.

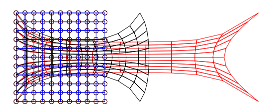

Figure 2: Compression deformation for force $-3e0$.

where the red-one is the solution with non-linear elasticity, the black-one with linear elasticity and the blue-one is the initial mesh. In both cases is observed that non-linearity has to be taken into account because there are large deformations. In the stretch case with non-linearity there is less deformation compared to the linear elasticity solution, otherwise, in the compression case with non-linearity there is more deformation. This observation is also noticed comparing the stress-strain curves:

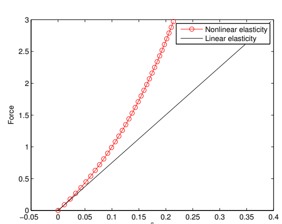

Figure 3: Stress-strain curve for force 3e0.

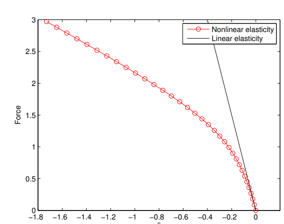

Figure 4: Stress-strain curve for force $-3e0$.

In both situations, the two curves are tangent close to zero displacement δ because the linear elasticity method relies on small deformations. Linear elasticity in this case must not be used because there are large deformations and it is observed that the stress-strain curves are not linear. There is no symmetry with respect to the sign of the loads as it is observed in the stress-strain curves.

1. In this case, instead of applying a force to the block, a displacement is imposed. In the first case is compressed until is reached half of the length and in the second case is stretched till doubling the length of the block.

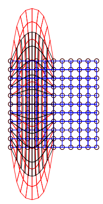

Figure 5: Compression deformation with $\delta = 0,5$.

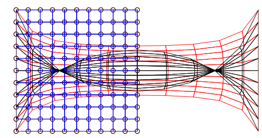

Figure 6: Stretch deformation with $\delta = 1$.

In the previous example, both forces give lower δ so in this example are obtained greater deformations than before as it is observed in Figure 6. In the stretching case seems that the Jacobian of some elements is negative, as it is shown in Figure 6, this is because the displacements have been exaggerated. According to the stress-strain curves

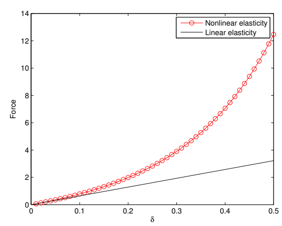

Figure 7: Stress-strain curve with $\delta = 0,5$.

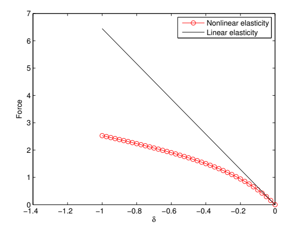

Figure 8: Stress-strain curve with $\delta = 1$.

as before, the two curves are tangent for small displacements for the same reason.

2. In the next two examples is studied a slender beam that is compressed. In the first case, where displacement is imposed, all the studied cases behave as the linear elasticity case despite the one that uses Newton-Raphson method with line-search and with random perturbations in the mesh. This shows that the linear solution is an unstable solution because changing a little bit the displacements at each step, the line-search method gives another solution.

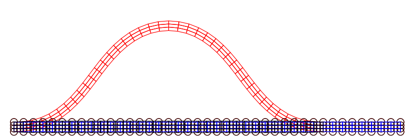

Figure 9: Deformation of example 2.

Also, the drawing of the stress-strain curve, shows that only with line-search this type of curve can be obtained because there is an abrupt change of trajectory in the stress-strain curve when buckling starts.

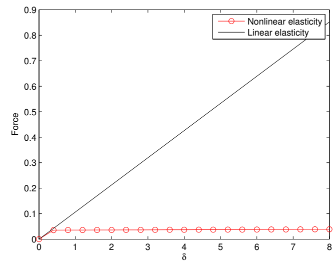

Figure 10: Stress-strain curve of example 2.

The graph also shows that when the beam begins to bend there is displacement without applying any force, while before starting to bend, the beams behaves linearly. Due to the symmetry of the problem, there is no reason to know in which direction the beam bends (see Figure 11) so half of the simulations give a different solution than the deformed in Figure 9, the solution is not unique.

3. The same occurs if, instead of imposing displacements in the beam, a force is imposed.

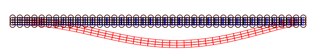

Figure 11: Deformation of example 3.

In the deformed is observed less buckling than the example before, but it also appears only with line-search and random perturbation of the mesh. In this case the beam bends at the other side of the beam, so the solution is not unique neither.

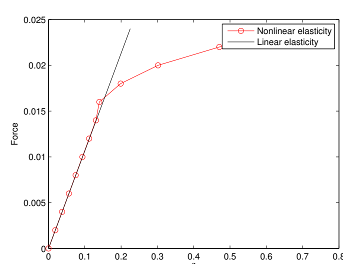

Figure 12: Stress-strain curve of example 3.

In the stress-strain curve it seems like a zoom of the previous example (see Figure 10) when buckling starts. Before the beam begins to bend, the stress-strain curve behaves like the linear elasticity curve.

4. Finally, the last two examples study the deformation of an arch. The first-one has the load at the center of the arch.

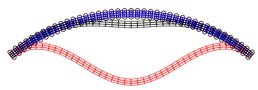

Figure 13: Deformation of example 4.

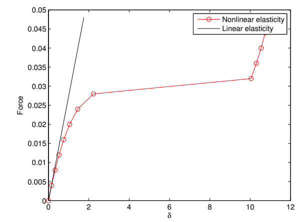

Figure 14: Stress-strain curve of example 4.

In the first example, with non-linear elasticity the arch collapses and it is obtained a deformed really far from the linear assumption. In the stress-strain curve it is observed that there is a point where the arch bends with really large displacements for little changes in the applied force. Then the arch returns to behave with a slope similar to the linear elasticity case. This curve can only be obtained using the Newton-Raphson method with line search.

5. In this example, the loads are applied close to the supports. The comparison of the deformed obtained is the following:

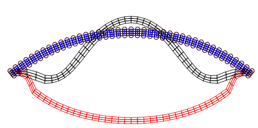

Figure 15: Deformation of example 5.

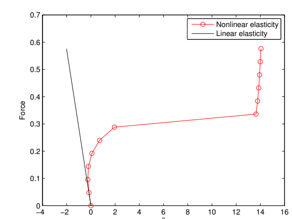

Figure 16: Stress-strain curve of example 5.

As in the previous example, the arch collapses and in the stress-strain curve there is a point where there are large displacements without changing a lot the value of the load.

In all the examples, at the beginning of the simulation both models look similar because there are small deformations but when there are larger deformations, they behave different. Also, buckling is a phenomenon that, in these examples, comes from an unstable equilibrium as seen in examples 2 and 3 so it is need a little vibration of the mesh to let the beam begin to bend. In some examples, the studied shape collapses and it is impossible to model it using the linear elasticity model and without line-search method.