

# NUMERICAL METHODS FOR PDE

MASTER'S DEGREE IN NUMERICAL METHODS IN ENGINEERING

# Assignment 4: Nonlinearity

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#### Abstract

In this assignment, it is required to calculate stresses on a steel plate with a hole, which is submitted to an axial tensile force. It is also done the modelling of the contact between a fixed pin and a plate, which is pulled at one of its ends. It is plotted the distribution of Von Mises stresses in each case, the force-displacement curve at a determined point-set and finally, it is introduced plastic properties to study the behaviour of the material then.

## 1 Task 1

#### 1.1 Von Mises stresses in the plate

Once all the steps of the tutorial have been done, it is obtained the distribution of Von Mises stresses in the plate shown in the figure 1.



Figure 1: Distribution of Von Mises stresses in the plate.

As it is shown, it exists an accumulation of stresses around the hole due to the fact it is a weak spot in the plate: stresses will concentrate on irregularities. The 460 MPa limit is overpassed because the maximal value is 1433 MPa.

#### **1.2** Force-Displacement curve

Next, it is plotted the relation between the force and the displacement of a node in the plate as it shown in 2.



Figure 2: Force(N)-Displacement(mm) curve.

As it is expected, it is shown a linear behaviour between force and displacement which represents the elasticity of the material. It is not applied any plasticity property yet.

#### **1.3** Plastic properties

It is modified the material properties to set plastic behaviour.

- Case A: Isotropic, make it perfectly plastic for  $f_y = 460$  N/mm2.
- Case B: Isotropic, with  $f_y = 460$ , plastic strain=0;  $f_{y2} = 520$ , plastic strain = 5e-3
- Case C: Isotropic, with  $f_y = 460$ , plastic strain = 0;  $f_{y2} = 520$ , plastic strain = 2e-3

First, the Von Mises stresses are shown in figure 3,4,5.



Figure 3: Von Mises stresses for case A.



Figure 4: Von Mises stresses for case B.



Figure 5: Von Mises stresses for case C.

In figure 6, it is presented the relationship between the displacement, in mm, and the force, in Newtons for each case.



Figure 6: Force(N)-Displacement(mm) curve for each case.

In the first case, it is implemented a perfectly plastic material and its behaviour is reflected in the curve. There is a bilinear curve (the little curve of the transition is neglected) which varies in a specified value. This is the definition of perfectly plastic, so the case can be considered as a validation of the model.

In the second case, it is defined as a second yield point which signifies that the slope will

decrease after the point specified. It is possible to see that the slope of the plastic part is not 0 as the perfectly plastic case but still is less than the elastic case.

However, in the third case, it is decreased the plastic strain where the variation occurs. It is also possible to see that the slope decreases progressively. The value of  $f_y = 460$  represents the transition from elastic to plastic on the curve. The plastic strain means irreversible displacement.

The theoretical behaviour for a specified material, with the elastic and plastic region until its fracture, is shown in figure 7.



Figure 7: Behaviour of a material

## 2 Task 2

#### 2.1 Von Mises stresses

In Figure 8, it is plotted the distribution of Von Mises stresses on the deformed shape with an amplification factor of 10. It is set a scale of stresses between 0-460 MPa and the stresses over this limit are plotted in dark red.

Due to the fact the pin is exerting force in the hole, there is a different behaviour. The distribution of stresses is reasonable because it is applied a force into the left part of the hole, therefore, it provokes more stresses in this side.



Figure 8: Distribution of Von Mises stresses.

### 2.2 Force-Displacement curve

Next, it is plotted the relationship between the force and the displacement of a node in the plate as it is shown in 9.



Figure 9: Force(N)-Displacement(mm) curve.

As it is expected, it is shown a linear behaviour between force and displacement which represents the elasticity of the material. It is not applied any plasticity property yet.

#### 2.3 Plastic properties

It is set the following properties for the plate material, the same for both cases:

- Isotropic, with  $f_y = 460$ , plastic strain=0;  $f_{y2} = 520$ , plastic strain = 5e-3. It is set the following properties for the pin material:
  - Case 1: Plastic, Isotropic,  $f_y = 900$ ,  $eps_p = 0$ .;  $f_y = 1000$ ,  $eps_p = 2e-3$ .
  - Case 2: Plastic, Isotropic,  $f_y = 320$ ,  $eps_p = 0$ .;  $f_y = 400$ ,  $eps_p = 5e-3$ .

In figures 10, 11 it is plotted the distribution of Von Mises for each case.



Figure 10: Distribution of Von Mises stresses for case 1.



Figure 11: Distribution of Von Mises stresses for case 2.

In figure 12, it is presented the relationship between the displacement, in mm, and the force, in Newtons for each case compared with the elastic case.



Figure 12: Force(N)-Displacement(mm) curve for each case..

In the first case, the pin is more resistant than the plate because it has a higher yielding strength as it is shown in figure 10. The plate is subjected to more stresses than the pin because it is the weaker entity. It is observed that in the sorrounding of the pin the stress is bigger tan 460 MPa. It is also noticed that the right part of the plat does not suffer a lot of stresses because all the action is happening on the left side of the pin.

In the second case, the pin is defined as the weaker part. It is noticed that the stresses are located in the plate in the contact zone. However, the stresses are under 460 MPa because the pin yields before the stress reaches this value and deforms afterwards.

In figure 12 is shown that the mechanical behaviour of the three case is the same before yielding. In the second case, it yields before the first case and the force keeps the lowest because the yield strengths are the smallest. In the first case it yields after the second and before the elastic case.

It is noticed that between 50-100 N it is located the transition from elastic to plastic behaviour. These values are much lower than the  $f_y$  parameters due to the non-linearity of the geometry.