Computational Mechanics Tools
Nonlinear Elastic Block

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1 Introduction

In this assignment, we experiment with the given nonlinear code for elasticity. We identify the code blocks that relate to importing the material properties, problem definition and solution methodology based on Newton Raphson and line search methods. We then run various examples of problems and discuss the results to compare the solutions obtained from linear and nonlinear analysis.

The main file for the code is `main_buckling.m` and `main_incremental_iterative.m`. Both codes implement the same methodology but the former includes the option to introduce initial geometric perturbation for the buckling analysis. This will become important for examples 2, 3, 4 and 5 which is concerning beams and archs. We will use the latter code for example 0 and 1 which only includes upsetting of a block.

2 Code identification

To identify the different sections in the code, we use `main_buckling.m`. It is more comprehensive as it includes the perturbations for buckling.

1. The main file for the code is `main_buckling.m`. Line 10 in this calls for the function `preprocess.m`. This function contains all the geometry details, material details for different problems cases. It accepts the example we are dealing with and the material to output details such as x and y limits of the problem, meshing of the domain with specified size along x and y directions, Gauss quadratures needed for integration, material properties and loading and boundary conditions (displacements).

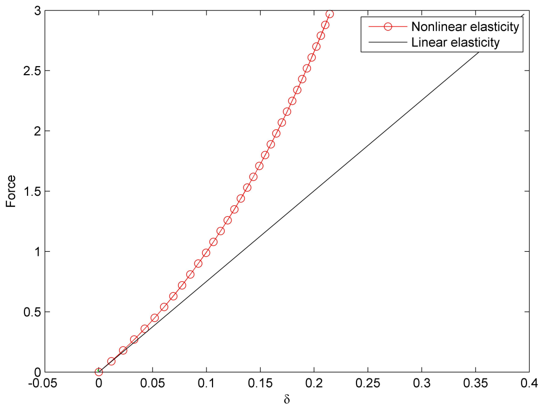
2. The choice of solution method is specified with *options.method* in line 18 and option to enable line search algorithm is found in line 20 of the file `main_buckling.m`

3. In `main_buckling.m`, the force is incremented in the for loop in line 68. For each force increment, the equilibrium position is calculated using the function `Equilibrate.m` in line 79. Here, the Newton Raphson method is implemented as case 1 in line 36 where the increment dx is calculated for until equilibrium is reached. It outputs the new equilibrium position.

4. Imposed displacements are applicable only for Case 1 and 2 in example. As mentioned earlier, the force is incremented in line 68 of `main_buckling.m`. For these two cases, the coordinates of the mesh are initially guessed in switch statements line 70 according to force increment.

5. Perturbations are introduced in the geometry for buckling cases in lines 54 to 56 in `main_buckling.m`. *mesh1.x0* is perturbed as a result. For random perturbations are introduced in line 77 with `x=x+rand(size(x))*.001` statement.

(a) force-displacement curve



(b) Deformed and undefromed meshes

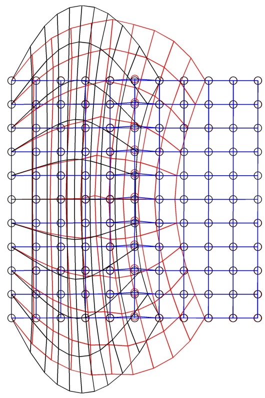
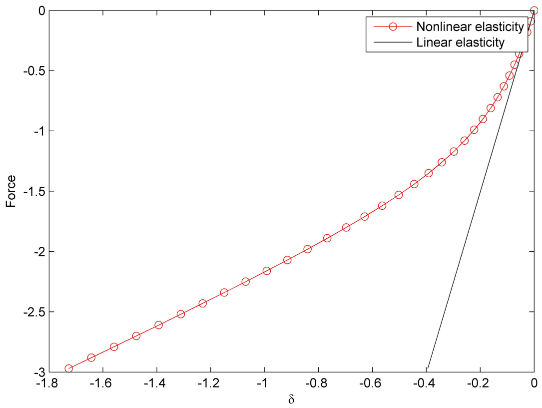


Figure 1: Compression force `mod1.force` $= -3$. In (b), undeformed mesh (blue), linear deformation (black) and nonlinear deformation (red)

(a) force-displacement curve



(b) Deformed and undefromed meshes

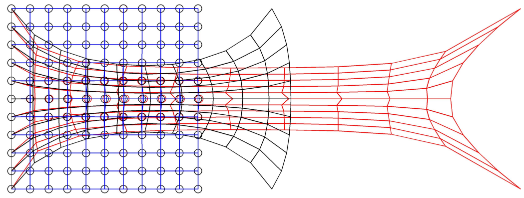


Figure 2: Tensile force `mod1.force` $= 3$. In (b), undeformed mesh (blue), linear deformation (black) and nonlinear deformation (red)

3 Test cases

In this section, we compare the linear and nonlinear behaviours of the elastic block, beam and arch for different test scenarios. We provide effect of introducing line-search algorithm to Newton Raphson and also the effect of perturbations, which are random in nature.

3.1 Example 0

In this case a square elastic block is subject to two different kinds of forces, one compressive and the other tensile. We do not employ line-search in this case and we present the result in fig1 and fig2.

Linear analysis makes several more assumptions than a nonlinear analysis. The distinguishing feature as a result is that linear analysis cannot capture the so called material and geometric nonlinearities. The geometric nonlinearity can be captured when we apply equilibrium equations in the deformed state and not restricting ourselves to small deformations. St. Venant's principle is also an assumptions of linear elasticity. The material nonlinearities are captured when the constitutive model is such that nonlinearities are permitted. These differences separate linear and nonlinear analysis.

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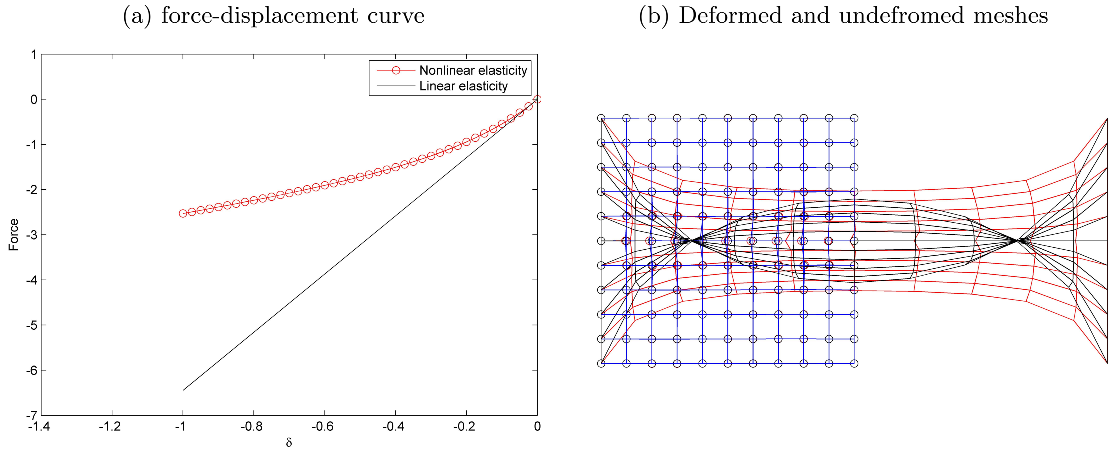

Figure 3: Tensile displacement until twice the original width. In (b), undeformed mesh (blue), linear deformation (black) and nonlinear deformation (red)

In fig 1, we can see that the linear behaviour is very close to nonlinear behaviour for very small displacements. But for large deformation, the assumption made by linear case (small deformation assumption) does not hold good. For compression, we can see that the linear case over-estimates the displacement for a given force. This is due to non-consideration to changes in area of cross section. But in tensile case 2, the linear analysis is conservative in its approximation and does not take into consideration the nonlinearities in material(such as yielding) and geometry nonlinearities(such as change in area of cross section). Hence it shows good agreement only for small displacements and for large forces, the displacement expected is much lower than that for a nonlinear study. This shows the drawbacks of a linear analysis.

We can observe that the behaviour of the material for compression and tension are the same for the linear analysis. They have the same slope in fig 1 and fig2. This shows that the linear study does not distinguish between the two behaviours because it is based on the original configuration of the block rather than on the current configuration.

3.2 Example 1

In this test case, the block is subject to a prescribed displacement. The displacement is tensile pull until twice the original width and compressive push until half the original width. They are depicted in fig3 and fig4 .

In fig 3, we see when nonlinearities are considered, the material deforms in stages, the change in area of the deformed body is taken into account. Thus we obtain a force-displacement diagram that shows us the variation of force required to cause the current deformation. Material shows higher resistance initially and then, for higher forces, the resistance to deformation is low. But the linear analysis predicts the same behaviour for small and large deformation. Hence it predicts a very large force for the displacement condition. Also the figure shows us two singularities in the block for linear analysis. Thus it predicts a failure even when its is not true. In fig4, the nonlinear material behaviour is captured where for small deformation, we expect small resistance and the resistance increases much faster for larger deformation. But the linear study disregards this and continues the trend of smaller force proportional to the deformation.

As commented in the previous section, we see the same behaviour in linear study for a tension and compression study (fig3 and fig4). While in nonlinear study, they are treated differently because it is based on the changing shape of the block.

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(a) force-displacement curve (b) Deformed and undefromed meshes

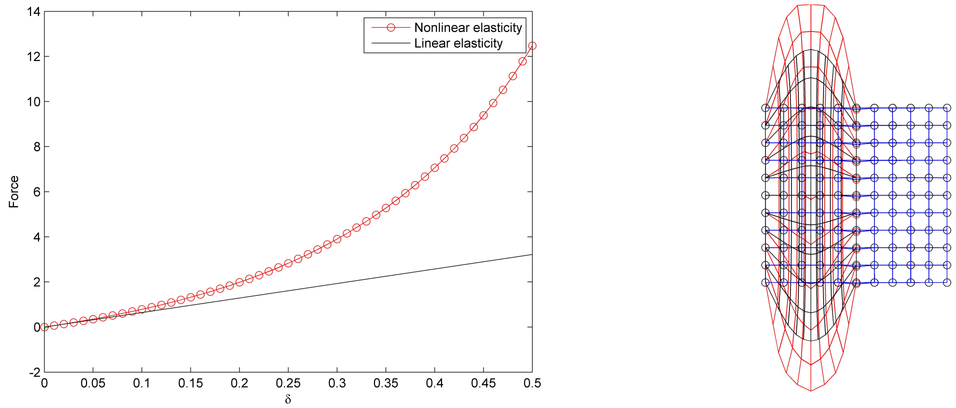

Figure 4: Compressive displacement until half the original width. In (b), undeformed mesh (blue), linear deformation (black) and nonlinear deformation (red)

3.3 Example 4 and 5

Here an arch problem is considered. Example 4 is related to an arch with dead load at the centre. Example 5 is an arch with dead load at the supports. The main idea to study the necessity of line-search in Newton Raphson. We would like to note that random perturbations are not necessary in the present case. Here, the solution does not depend on perturbations since there can be no 'upsetting' due to additional random irregularities in the position. It is confirmed by the simulation too. We do not observe any change in behaviour when the perturbations are present or absent.

Line-search with Newton Raphson eliminates the unstable equilibrium points and chooses local minima that is stable. Saddle points and maxima are thus eliminated and stable results are obtained. This is because Newton Raphson method does not distinguish between different kinds of local maxima or minima. Thus line-searchi is a necessary tool in such cases.

We observe that for example 4, when line-search was not used, the random perturbations caused the solution to diverge and equilibrium was not obtained. But when line search was utilized, we were able to observe convergence in the solution error and thus a stable solution was obtained as shown in fig5 .

We can observe in fig5 that the linear behaviour is very different from the nonlinear behaviour. In the nonlinear behaviour, we observe a quick jump in the displacement when the load is increased around 0.03. This is a 'snapping mechanism' where the arch observe a sudden collapse and large deformation for very small increase in load. This is not necessarily a bad phenomenon. In fact, such nonlinear behaviours are used in many applications such as a snapping mechanism in opening and closing of bottle cap, snapping hair clips etc. Such behaviour is completely ignored by the linear analysis. The deformed mesh give us an idea of the large discrepancy in the final deformation predicted by both analyses.

The above nonlinear behaviour is only captured and stabilitiy is restored due to the line-search algorithm that selects the suitable stable equilibrium point.

We observed a similar situation for example 5 where a line-search with Newton Raphson was necessary for convergence of error and to obtain a stable solution as shown in fig6. Without line-search we obtained an unstable, meaningless solution.

The nonlinear study predicts the 'snapping behaviour' as in the previous example. This is not captured by the linear study. It is thus inferior in simulating real physics in the phenomenon.

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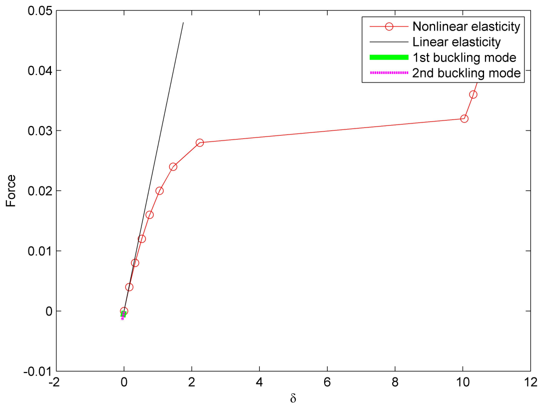
(a) force-displacement curve



(b) Deformed and undeformed meshes

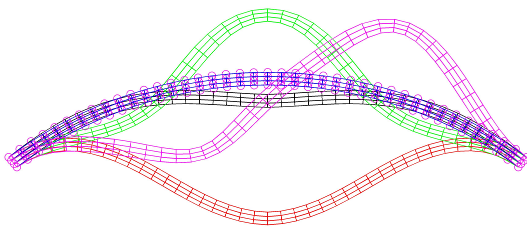
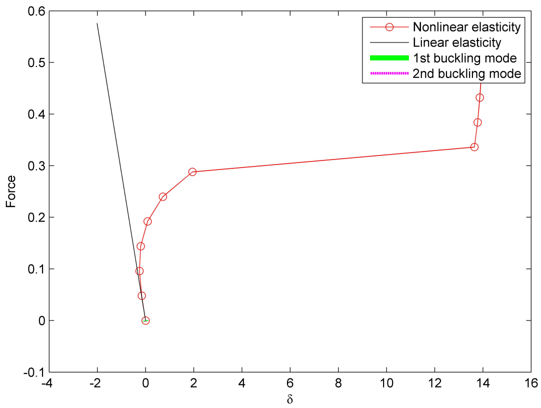


Figure 5: Stable solution of Example 4 using line-search. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

(a) force-displacement curve



(b) Deformed and undeformed meshes

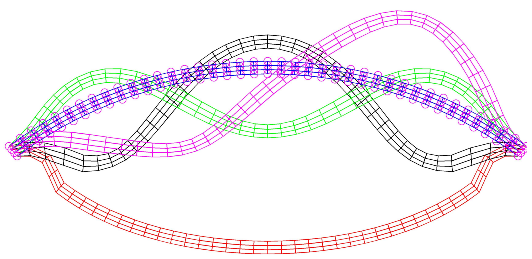


Figure 6: Stable solution of Example 5 using line-search. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

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(a) force-displacement curve (b) Deformed and undeformed meshes

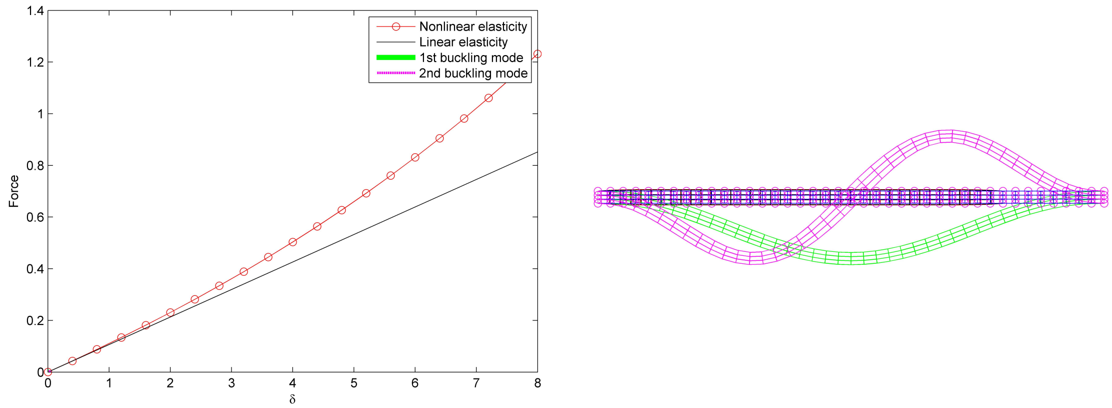

Figure 7: Example 4 **(no linesearch, no perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

(a) force-displacement curve (b) Deformed and undeformed meshes

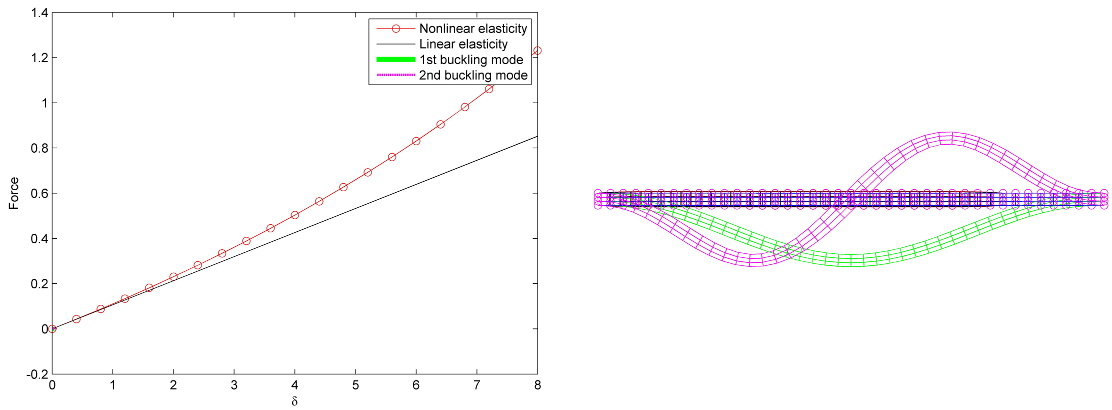

Figure 8: Example 4 **(no linesearch, with perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

4 Example 2 and 3

In this section we look at example 2 which deals with a slender beam subject to prescribed displacement and at example 3 which deals with a slender beam subject to a dead load. We shall analyse the problem in the presence of random perturbations and the effect of use of line-search in Newton Raphson. The summary of result are presented in the table below. 'ls' denotes line-search and 'per' denotes perturbation.

	ls(no) per(no)	ls(no) per(yes)	ls(yes) per(no)	ls(yes) per(yes)
Example 2	no buckling	no buckling	no buckling	buckling
Example 3	no buckling	buckling	buckling	buckling

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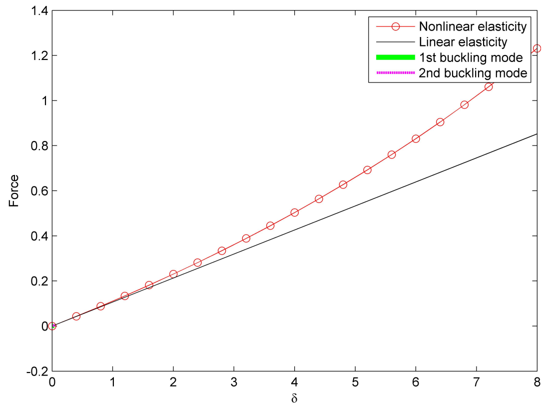
(a) force-displacement curve



(b) Deformed and undeformed meshes

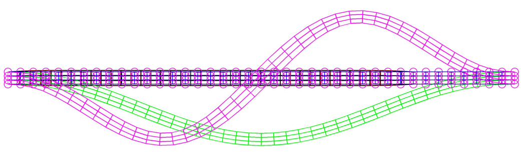
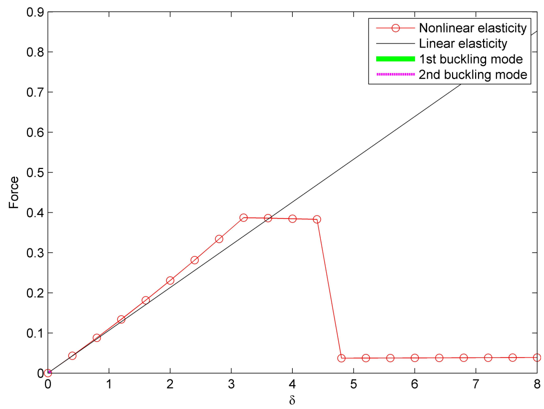


Figure 9: Example 4 **(with linesearch, no perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

(a) force-displacement curve



(b) Deformed and undeformed meshes

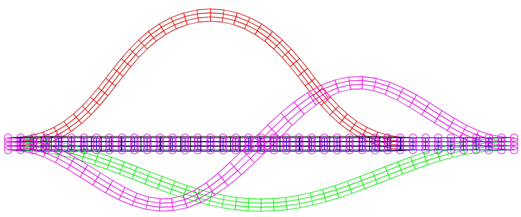


Figure 10: Example 4 **(with line search, with perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

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Example 2 and 3 show the importance of perturbations for problems with symmetric loading conditions.

In example 2, the slender beam is subject to prescribed displacement and we see different cases presented in fig7, fig8, fig9 and fig10. When either perturbations or line-search are not employed, we do not see a drastic nonlinear behaviour (fig7, fig8 and fig9). The nonlinear behaviour is close to linear behaviour for small deformations as expected. This is the case of the beam moves along its axis. This is an ideal nonlinear behaviour and all the cases provide the same solution to the problem.

Cases such as above occur when there are absolutely no forces perpendicular to the beam. But they are not realistic. In reality, when axial displacements are enforced, small fluctuations are observed in the perpendicular direction and this is the trigger for buckling. These random perturbations are a necessary part of realistic modelling and when introduced with line search algorithm to capture the stable equilibrium, we obtain buckling as shown in fig10. Without line-search, this is not captured (fig8). This shows us that perturbation tend to destabilize the solution and line search tends to restore stability and both are necessary to obtain nonlinear behaviour in the presence of no external disturbance in the solution domain. (We can observe the same for classical flow cases such as flow past a cylinder. To observe Von Karman vortices, we need certain initial perturbation in velocity field or slightly unsymmetric boundary conditions. The latter is applied by placing the cylinder at a slightly different distance to the walls on either sides. In the absence of disturbance, we observe a flow similar to incompressible flow past a cylinder)

For example 3, the slender beam is subject to axial forces in the form of dead load and we see different cases presented in fig11, fig12, fig13 and fig14. The problem contains inherent source of fluctuation and it is easy to obtain the buckling if either line-search or random pertrubations are employed. This is a realistic simulation (fig12, fig13 and fig14,) since they all agree with respect to the force-displacement curve. Here, either the random fluctuation is sufficient to drive the solution towards buckling nonlinear behaviour in the presence or absence of line-search. Consequently, line-search is sufficient to detect and capture the stable equilibrium and arrive at a buckling solution in the presence or absence of random fluctuations. But, when both are absent(fig11), we do not observe any nonlinearity. It is exactly depicted by the linear behaviour.

We can observe that in fig12 and fig13, we obtain the same force-displacement graphs, we do not see buckling in the same side. This shows that such behaviour are inherently non-unique. This is because the fluctuations inflicting the change are random in nature.

Note on magnitude of random perturbations: We noticed that when the perturbations are below certain value, it did not trigger the nonlinear behaviour. Hence a small amount of perturbations is necessary for destabilizing the solution such that the stability can be then restored by line-search. But above a certain value of perturbation, the system lost the stability totally and couldn't be restored. Thus it was advisible to work within a small window of perturbation, which is lies between the above said minimum and maximum. Also we need to note that for any value within this range of perturbations, no significant variation in solution was observed. Thus perturbations are necessary components for small destabilization which needs to be restored and this leads to observe nonlinear behaviour.

5 Conclusions

In analysing example 0 and 1 we learnt that linear study is insufficient and is only valid for small deformations. At large deformation, we obtain a nonrealistic result. But a non-linear solution captures behaviour for all values of deformation.

Linear study gives symmetric results for opposite boundary conditions. It cannot distinguish between compression and tension other than the sign. It doesn't take into account material and geometric nonlinearities and predicts a similar behaviour. The slopes of the force-displacement curves are the same in tension and compression. But on the other hand, nonlinear behaviour is different for these two phenomenon and the force-displacement curves are hence different.

In example 4 and 5, we notice that the 'snapping behaviour' captured in the nonlinear analysis

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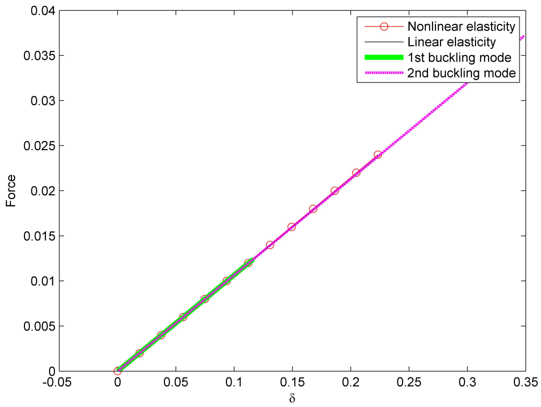
(a) force-displacement curve



(b) Deformed and undeformed meshes

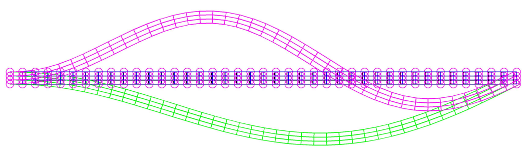
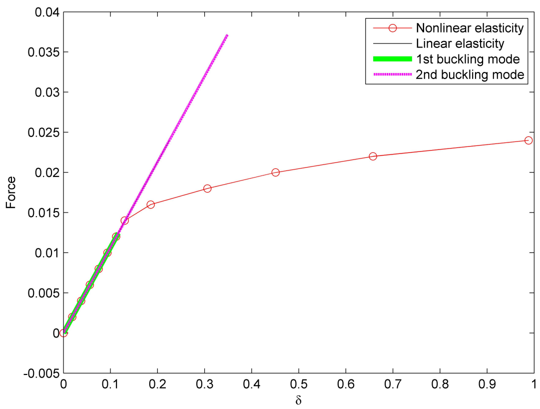


Figure 11: Example 5 **(no linesearch, no perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

(a) force-displacement curve



(b) Deformed and undeformed meshes

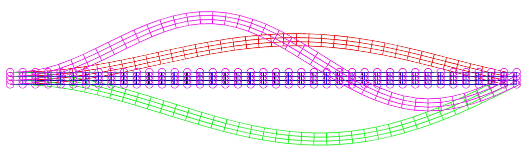


Figure 12: Example 5 **(no linesearch, with perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

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(a) force-displacement curve (b) Deformed and undeformed meshes

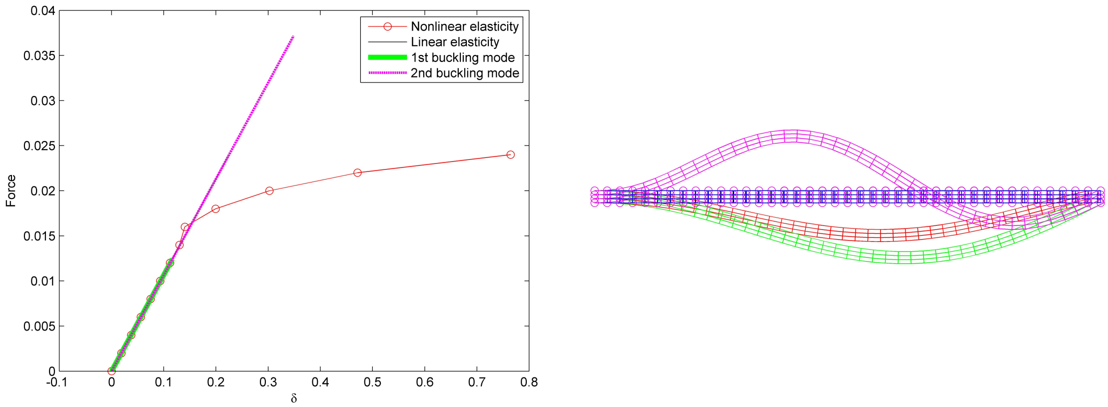

Figure 13: Example 5 **(with linesearch, no perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

(a) force-displacement curve (b) Deformed and undeformed meshes

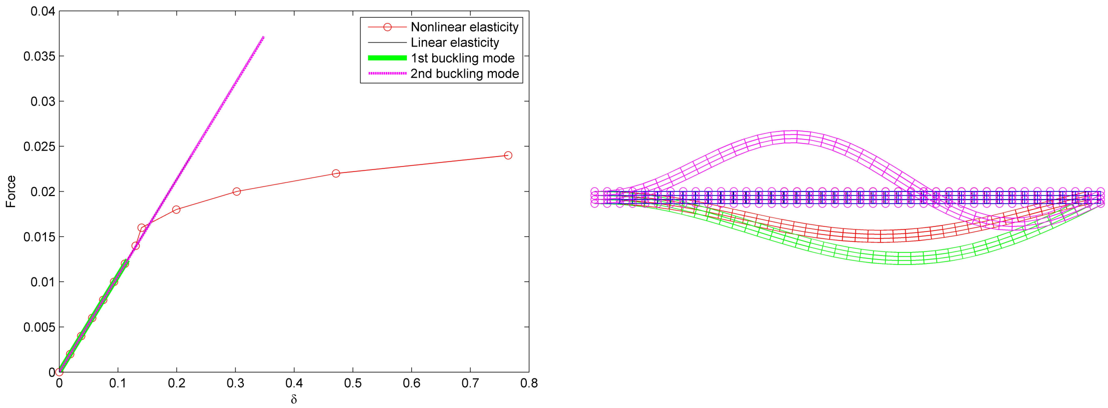

Figure 14: Example 5 **(with linesearch, with perturbation)**. In (b), undeformed mesh (blue), linear deformation (black), nonlinear deformation (red), first buckling mode (green) and second buckling mode (magenta)

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is absent in linear analysis. It also indicates how line search algorithm is necessary for Newton Raphson method to recognise stable points and obtain equilibrium nonlinear solution even in the absence of random perturbations. For above cases, since there were no perturbations, and the problem is not susceptible to random variations in solution, we observe stable and unique solution.

In example 2 and 3, we learnt that perturbations are important to bring out the nonlinear behaviour of materials, otherwise we will only notice the behaviour predicted by linear analysis. For problems with symmetric boundary conditions we observed that perturbations are necessary to destabilize the system and line-search is necessary to restore the stability. In the above process, the nonlinear behaviour of buckling was captured. We discussed an analogous behaviour in flow past a cylinder. Also we observed that for problems with inherent instability, either random perturbations or line-search was sufficient to produce the nonlinear behaviour. Perturbation, we observed, will not necessarily lead to unique solutions.

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