THE PHYSICS BEHIND BOATS AND PLANES

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Introduction



Hydrostatics - Review

- Floats if:
 Equilibrium between weight and buoyancy.
- Stable if: $GM \gg 0$ & $V_{lost} = V_{gained}$ & $\frac{d(G)}{d\theta} = 0$



Stability – Righting arm



The ship will keep rolling and may sink.

Stability – Righting arm



$RM = GZ \cdot \Delta$

 $GZ = GM \cdot sin\phi$

*This is valid only for small angles

Stability

• Old fashion way:

$$\begin{split} RM &= GZ \cdot \Delta \\ GZ &= GM \cdot sin\phi \end{split}$$

Valid if
$$V_{lost} = V_{gained}$$



- Modern techniques:
 numerical method
 computational aid
- G and B for any angle
- CoG may not be constant



How is lift generated? – Review

Lift is generated by changing the velocity of a flow.



2D thin airfoil theory – Potential flow

- Simple solution for thin airfoils: neglect thickness effects and use a mean-line model.
- Consider the airfoil as a distribution of vortices along the mean line:

$$\gamma(\theta) = 2V_{\infty}\left(A_0 \tan(\theta/2) + \sum_{n=1}^{\infty} A_n \sin(n\theta)\right)$$

2D thin airfoil theory – Potential flow

- Kutta condition: zero vorticity at the trailing edge
- Boundary condition: no circulation in the normal direction and $u_i \ll v_i$

$$V_n = 0 \rightarrow \frac{v}{u} = \frac{V_\infty \sin(\alpha) + v_i}{V_\infty \cos(\alpha) + u_i} = \frac{dy}{dx} \approx \alpha + \frac{v_i}{V_\infty}$$

• For an irrotational vortex:

$$v_i = \frac{\gamma}{2\pi r} = -V_{\infty}(A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta))$$

2D thin airfoil theory – Potential flow

• Then solve for the coefficients A in the equation:

$$\frac{dy}{dx} = \alpha - A_0 - \sum_{n=1}^{\infty} A_n \cos(n\theta)$$

 With all the coefficients, the lift of a small element of the vortex line can be predicted and the total lift generated will be:

$$Lift = \rho V_{\infty} \sum d\Gamma \quad ; \quad C_{\rm L} = \frac{Lift}{\frac{1}{2}\rho V_{\infty}^2 c}$$

http://dhaerotech.com/Images/Clark%20Y%20Aileron%20Forces.jpg

Factors that affect lift

- Those associated with the **object**: shape and size.
- Those associated with the **motion of the object** through the air: velocity and inclination.
- Those associated with the **air** itself: mass, viscosity, compressibility.

All of them are gathered in the lift equation: $\ L=rac{1}{2}
ho V^2AC_L$

Shape and size effects

- Lift is generally a complex function of the shape.
- Shape effect modelled by the lift coefficient.
- Lift depends specially on the amount of camber. Camber $\uparrow \rightarrow$ Lift \uparrow
- Lift is proportional to the area. Doubling the area → doubling the lift



Velocity, density and inclination effects

- Velocity: relative velocity between the object and the flow. Doubling the velocity → quadrupling the lift
- Density: Linear dependence. Halving the density → Halving the lift
- Angle of attack: angle between the chord line and the flight direction. Large effect on the lift.

