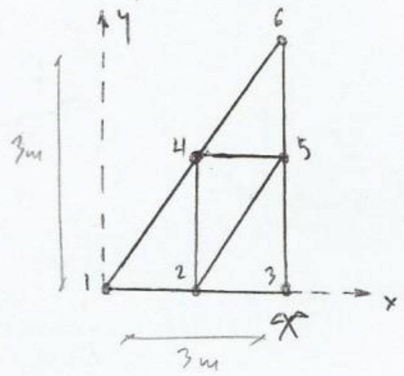
1.- Strong Form:



$$EA \frac{\partial^2 u}{\partial x^2} + EA\frac{\partial^2 u}{\partial y^2} = 0$$

$t = 1$

$$E(3-y)t \frac{\partial^2 u}{\partial x^2} + Ext\frac{\partial^2 u}{\partial y^2} = 0$$

$$\boxed{E(3-y) \frac{\partial^2 u}{\partial x^2} + Ex\frac{\partial^2 u}{\partial y^2} = 0}$$

1.-

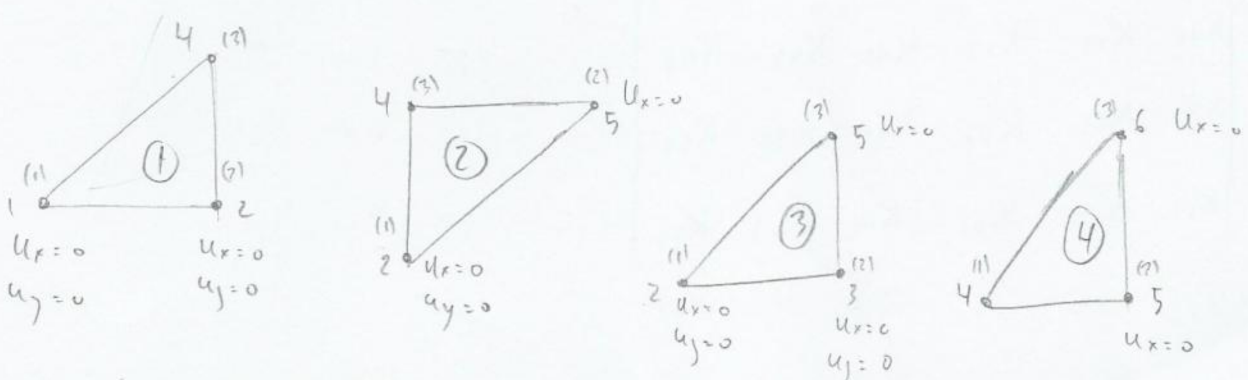
Condiciones de contorno

- Los nodos ①, ②, ③ no tienen desplazamiento $u=0$ en 'x' y en 'y' al estar empotrados $\boxed{u(x, y=0) = 0}$
- El desplazamiento horizontal de los nodos ③, ⑤, ⑥ es cero $u_x = 0$ ya que el desplazamiento impuesto y el peso actua verticalmente $\boxed{\frac{\partial u}{\partial x}(3, y) = 0}$

2.-

Nodal Coordinates → $X = \begin{bmatrix} 0 & 0 \\ 1{,}5 & 0 \\ 3 & 0 \\ 1{,}5 & 1{,}5 \\ 3 & 1{,}5 \\ 3 & 3 \end{bmatrix}$ Connectivity Matrix → $T = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

3.-



$\boxed{\text{4 Grados de Libertad} \quad \text{nodo 4} - u_{x4},\ u_{y4} \quad \text{nodo 5} - u_{y5} \quad \text{nodo 6} - u_{y6}}$

Matrices de rigidez locales

$$K^{(e)} = B^T D B t \quad \text{para } t=1 \rightarrow K^{(e)} = B^T D B$$

$$B = \frac{1}{2A^e}\begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix}$$

(Plane stress) $D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu^2)}{2(1+\nu)} \end{bmatrix}$

$A = \frac{1,5 \cdot 1,5}{2} = 1,125$ Para todos los elementos

Para elementos 1, 3, 4

$$\begin{cases} b_1 = y_2 - y_3 = -1,5 \\ b_2 = y_3 - y_1 = 1,5 \\ b_3 = y_1 - y_2 = 0 \end{cases}$$

$$\begin{cases} c_1 = x_3 - x_2 = 1,5 - 1,5 = 0 \\ c_2 = x_1 - x_3 = 0 - 1,5 = -1,5 \\ c_3 = x_2 - x_1 = 1,5 - 0 = 1,5 \end{cases}$$

Para elemento 2

$$\begin{cases} b_1 = 0 \\ b_2 = 1,5 \\ b_3 = -1,5 \end{cases}$$

$$\begin{cases} c_1 = -1,5 \\ c_2 = 0 \\ c_3 = 1,5 \end{cases}$$

$$B^{(1,3,4)} = \frac{1}{2 \cdot 1,125}\begin{bmatrix} -1,5 & 0 & 1,5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1,5 & 0 & 1,5 \\ 0 & -1,5 & -1,5 & 1,5 & 1,5 & 0 \end{bmatrix}$$

$$B^{3} = \frac{1}{2 \cdot 1,125}\begin{bmatrix} 0 & 0 & 1,5 & 0 & -1,5 & 0 \\ 0 & -1,5 & 0 & 0 & 0 & 1,5 \\ -1,5 & 0 & 0 & 1,5 & 1,5 & -1,5 \end{bmatrix}$$

$$K^{(e)}_{local} = \begin{array}{c} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \end{array} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \\ u_{x1} & u_{x2} & u_{x2} & u_{x4} \\ u_{y1} & u_{y2} & u_{y2} & u_{y4} \\ u_{x2} & u_{x5} & u_{x3} & u_{x5} \\ u_{y2} & u_{y5} & u_{y3} & u_{y5} \\ u_{x4} & u_{x4} & u_{x5} & u_{x6} \\ u_{y4} & u_{y4} & u_{y5} & u_{y6} \end{array}$$

$e_1 \rightarrow u_{x1} \quad u_{y1} \quad u_{x2} \quad u_{y2} \quad u_{x4} \quad u_{y4}$

$e_2 \rightarrow u_{x2} \quad u_{y2} \quad u_{x5} \quad u_{y5} \quad u_{x4} \quad u_{y4}$

$e_3 \rightarrow u_{x2} \quad u_{y2} \quad u_{x3} \quad u_{y3} \quad u_{x5} \quad u_{y5}$

$e_4 \rightarrow u_{x4} \quad u_{y4} \quad u_{x5} \quad u_{y5} \quad u_{x6} \quad u_{y6}$

$$K = \begin{bmatrix}
K_{11}^{(1)} & K_{12}^{(1)} & K_{13}^{(1)} & K_{14}^{(1)} & 0 & 0 & K_{15}^{(1)} & K_{16}^{(1)} & 0 & 0 & 0 & 0 \\
 & K_{22}^{(1)} & K_{23}^{(1)} & K_{24}^{(1)} & 0 & 0 & K_{25}^{(1)} & K_{26}^{(1)} & 0 & 0 & 0 & 0 \\
 & & K_{33}^{(1)}+K_{11}^{(2)}+K_{11}^{(3)} & K_{34}^{(1)}+K_{12}^{(2)}+K_{12}^{(3)} & K_{13}^{(3)} & K_{14}^{(3)} & K_{35}^{(1)}+K_{15}^{(2)} & K_{36}^{(1)}+K_{16}^{(2)} & K_{13}^{(2)}+K_{15}^{(3)} & K_{14}^{(2)}+K_{16}^{(3)} & 0 & 0 \\
 & & & K_{44}^{(1)}+K_{22}^{(2)}+K_{22}^{(3)} & K_{23}^{(3)} & K_{24}^{(3)} & K_{45}^{(1)}+K_{25}^{(2)} & K_{46}^{(1)}+K_{26}^{(2)} & K_{23}^{(2)}+K_{25}^{(3)} & K_{44}^{(2)}+K_{66}^{(3)} & 0 & 0 \\
 & & & & K_{33}^{(3)} & K_{34}^{(3)} & 0 & 0 & K_{35}^{(3)} & K_{36}^{(3)} & 0 & 0 \\
 & & & & & K_{44}^{(3)} & 0 & 0 & K_{45}^{(3)} & K_{46}^{(3)} & 0 & 0 \\
 & & \text{Simetrica} & & & & K_{55}^{(1)}+K_{55}^{(2)}+K_{11}^{(4)} & K_{56}^{(1)}+K_{56}^{(2)}+K_{12}^{(4)} & K_{53}^{(2)}+K_{13}^{(4)} & K_{54}^{(2)}+K_{14}^{(4)} & K_{15}^{(4)} & K_{16}^{(4)} \\
 & & & & & & & K_{66}^{(1)}+K_{66}^{(2)}+K_{22}^{(4)} & K_{63}^{(2)}+K_{23}^{(4)} & K_{64}^{(2)}+K_{24}^{(4)} & K_{25}^{(4)} & K_{26}^{(4)} \\
 & & & & & & & & K_{33}^{(3)}+K_{33}^{(4)} & K_{34}^{(2)}+K_{34}^{(4)} & K_{35}^{(4)} & K_{36}^{(4)} \\
 & & & & & & & & & K_{44}^{(2)}+K_{44}^{(4)} & K_{45}^{(4)} & K_{46}^{(4)} \\
 & & & & & & & & & & K_{55}^{(1)} & K_{56}^{(4)} \\
 & & & & & & & & & & & K_{66}^{(4)}
\end{bmatrix}$$

El vector desplazamientos

$$u = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \\ u_{x6} \\ u_{y6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u_{x4} \\ u_{y4} \\ 0 \\ u_{y5} \\ 0 \\ \delta \end{bmatrix}$$

El vector fuerza

$$f = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \\ f_{5x} \\ f_{5y} \\ f_{6x} \\ f_{6y} \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} + fb_y^{(1)} \\ R_{2x} \\ R_{2y} + fb_y^{(1)} + fb_y^{(2)} + fb_y^{(4)} \\ R_{3x} \\ R_{3y} + fb_y^{(3)} \\ 0 \\ fb_y^{(1)} + fb_y^{(2)} + fb_y^{(4)} \\ 0 \\ fb_y^{(4)} + fb_y^{(2)} + fb_y^{(3)} \\ 0 \\ fb_y^{(4)} \end{bmatrix}$$

Siendo $fb_y^{(1)} = fb_y^{(2)} = fb_y^{(3)} = fb_y^{(4)}$

$$= \frac{A \cdot t}{3} \cdot b \cdot \gamma \begin{Bmatrix} b_x \\ b_y \end{Bmatrix} = \begin{matrix} 0 \\ 375 \end{matrix}$$

Al sólo haber 4 grados de libertad, la matriz global se puede simplificar haciendo cero las columnas correspondiente a los desplazamientos impedidos. De esta forma

$$K = \left[\begin{array}{ccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1,0417 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -2,0833 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0 & 0 & -4,1666 & 3,125 & 0 & -3,125 & 0 & 0 & 0 \\
 & & & 0 & 0 & 0 & 3,125 & -10,416 & 0 & 12,5 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 & 0 & 1,0417 & 0 & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & -5,2083 & 0 & 0 & 0 \\
 & & & & & & 14,583 & -3,125 & 0 & 2,0833 & 0 & 0 & 0 \\
 & & & & & & & 14,587 & 0 & -4,1666 & 0 & 0 & 0 \\
 & \text{Simetrica} & & & & & & & 0 & -3,125 & 0 & 0 & 0 \\
 & & & & & & & & & & 9,375 & 0 & -5,2083 \\
 & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 5,2083
\end{array}\right] \cdot 1 \cdot 10^9$$

Las matrices K^1 K^2 K^3 y K^4 han sido calculadas con Matlab en el archivo adjunto.

Para $D = 1 \cdot 10^{10} \begin{bmatrix} 1,0417 & 0,2083 & 0 \\ 0,2083 & 1,0417 & 0 \\ 0 & 0 & 0,4167 \end{bmatrix}$

4.

El sistema de ecuaciones a resolver es:

$$\begin{bmatrix} \\ \\ K_{global} \\ \\ \\ \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u_{x4} \\ u_{y4} \\ 0 \\ u_{y5} \\ 0 \\ \delta \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} + fb_y^{(1)} \\ R_{2x} \\ R_{2y} + fb_y^{(1)} + fb_y^{(2)} + fb_y^{(3)} \\ R_{3x} \\ R_{3y} + fb_y^{(3)} \\ 0 \\ fb_y^{(1)} + fb_y^{(2)} + fb_y^{(4)} \\ 0 \\ fb_y^{(4)} + fb_y^{(2)} + fb_y^{(3)} \\ 0 \\ fb_y^{(4)} \end{bmatrix}$$

12 ecuaciones

10 incognitas

```
>> B1

B1 =

   -0.6667         0    0.6667         0         0         0
         0         0         0   -0.6667         0    0.6667
         0   -0.6667   -0.6667    0.6667    0.6667         0

>> K1

K1 =

   1.0e+09 *

    5.2083         0   -5.2083    1.0417         0   -1.0417
         0    2.0833    2.0833   -2.0833   -2.0833         0
   -5.2083    2.0833    7.2917   -3.1250   -2.0833    1.0417
    1.0417   -2.0833   -3.1250    7.2917    2.0833   -5.2083
         0   -2.0833   -2.0833    2.0833    2.0833         0
   -1.0417         0    1.0417   -5.2083         0    5.2083

>> B2

B2 =

         0         0    0.6667         0   -0.6667         0
         0   -0.6667         0         0         0    0.6667
   -0.6667         0         0    0.6667    0.6667   -0.6667

>> K2

K2 =

   1.0e+09 *

    2.0833         0         0   -2.0833   -2.0833    2.0833
         0    5.2083   -1.0417         0    1.0417   -5.2083
         0   -1.0417    5.2083         0   -5.2083    1.0417
   -2.0833         0         0    2.0833    2.0833   -2.0833
   -2.0833    1.0417   -5.2083    2.0833    7.2917   -3.1250
    2.0833   -5.2083    1.0417   -2.0833   -3.1250    7.2917
```

```
>> B3

B3 =

   -0.6667         0    0.6667         0         0         0
         0         0         0   -0.6667         0    0.6667
         0   -0.6667   -0.6667    0.6667    0.6667         0

>> K3

K3 =

   1.0e+09 *

    5.2083         0   -5.2083    1.0417         0   -1.0417
         0    2.0833    2.0833   -2.0833   -2.0833         0
   -5.2083    2.0833    7.2917   -3.1250   -2.0833    1.0417
    1.0417   -2.0833   -3.1250    7.2917    2.0833   -5.2083
         0   -2.0833   -2.0833    2.0833    2.0833         0
   -1.0417         0    1.0417   -5.2083         0    5.2083

>> B4

B4 =

   -0.6667         0    0.6667         0         0         0
         0         0         0   -0.6667         0    0.6667
         0   -0.6667   -0.6667    0.6667    0.6667         0

>> K4

K4 =

   1.0e+09 *

    5.2083         0   -5.2083    1.0417         0   -1.0417
         0    2.0833    2.0833   -2.0833   -2.0833         0
   -5.2083    2.0833    7.2917   -3.1250   -2.0833    1.0417
    1.0417   -2.0833   -3.1250    7.2917    2.0833   -5.2083
         0   -2.0833   -2.0833    2.0833    2.0833         0
   -1.0417         0    1.0417   -5.2083         0    5.2083
```