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HOMEWORK #2

1. **Define the strong form of the problem in the reduced domain (left hand). Indicate accurately the Boundary Conditions in every edge.**

   Using the linear momentum equilibrium equation:

$$Div\,\sigma + \rho b = 0$$

   Where the stress and strain values are defined in the next way:

$$\sigma = D \cdot \varepsilon \qquad \varepsilon_x = \frac{du}{dx} \qquad \varepsilon_y = \frac{dv}{dy}$$

$$D = \begin{pmatrix} d_{11} & d_{12} & 0 \\ d_{12} & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix} \quad ,$$

$$d_{11} = d_{22} = \frac{E}{1-\mathrm{v}^2} \quad , \quad d_{12} = d_{21} = \mathrm{v}\frac{E}{1-\mathrm{v}^2} \quad , \quad d_{33} = \frac{E}{2\cdot(1+\mathrm{v})}$$

   And b is the vector with the forces acting on the plate. Note that the strong form is a 2nd order derivate problem with the next boundary conditions:

   Defining the displacement's vector a(u,v):

$$\begin{aligned} &v_6 = \delta \\ &u_1 = u_2 = u_3 = u_5 = u_6 = 0 \\ &v_1 = v_2 = v_3 = 0 \end{aligned}$$

2. **Describe the mesh shown in figure 2 by giving the arrays of nodal coordinates X and the connectivity matrix T. In order to simplify the computations select the local numbering of the nodes such that, in every element, the node in the right angle vertex has local number equal to 1.**

   In the next table the mesh is describe, we can see the local numbering used and its global numbering associated, also its x-y coordinates in the global system. Also in the left-column, we can see how the nodes are connected:

Element	**Node**		**Connectivities**
	Local	**Global (x,y)**	
1	1	2 (1.5,0)	**1,3,4,5**
	2	4(1.5,1.5)	**1,2,3,5,6**
	3	1(0,0)	**2,4**
2	1	4(1.5,1.5)	**1,2,4,5,6**
	2	2(1.5,0)	**2,4,5**
	3	5(3,1.5)	**2,3,4,5,6**
3	1	3(3,0)	**2,5**
	2	5(3,1.5)	**2,3,4,5,6**

	3	2(1.5,0)	**1,3,4,5**
4	1	5(3,1.5)	**2,3,4,5,6**
	2	6(3,3)	**4,5**
	3	4(1.5,1.5)	**1,2,3,5,6**

3. **Set up the linear system of equations corresponding to the discretization in figure 2. How many degrees of freedom has the system to be solved?**

The linear system to be solved has to be of the form:

$$K \cdot a = F$$

Where K is the stiffness matrix, which results from assembling the different element stiffness matrices $K_{ij}^{(e)}$ and is symmetric:

$$\begin{bmatrix} K_{33}^{(1)} & K_{31}^{(1)} & 0 & K_{23}^{(1)} & 0 & 0 \\ K_{31}^{(1)} & K_{11}^{(1)} + K_{22}^{(2)} + K_{33}^{(3)} & K_{31}^{(3)} & K_{12}^{(1)} + K_{12}^{(2)} & K_{23}^{(2)} + K_{23}^{(3)} & 0 \\ 0 & K_{31}^{(3)} & K_{11}^{(3)} & 0 & K_{12}^{(3)} & 0 \\ K_{23}^{(1)} & K_{12}^{(1)} + K_{12}^{(2)} & 0 & K_{22}^{(1)} + K_{11}^{(2)} + K_{33}^{(4)} & K_{13}^{(2)} + K_{31}^{(4)} & K_{23}^{(4)} \\ 0 & K_{23}^{(2)} + K_{23}^{(3)} & K_{12}^{(3)} & K_{13}^{(2)} + K_{31}^{(4)} & K_{33}^{(2)} + K_{22}^{(3)} + K_{33}^{(4)} & K_{12}^{(4)} \\ 0 & 0 & 0 & K_{23}^{(4)} & K_{12}^{(4)} & K_{22}^{(4)} \end{bmatrix}$$

$$\cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} r_1 + f_b^{(1)} \\ r_2 + f_b^{(1)} + f_b^{(2)} \\ r_3 + f_b^{(3)} \\ f_b^{(1)} + f_b^{(2)} + f_b^{(3)} \\ f_b^{(2)} + f_b^{(3)} + f_b^{(4)} + r_5 \\ f_b^{(4)} + f_\varepsilon^{(4)} \end{bmatrix}$$

Where the $K_{ij}^{(e)}$ matrix are defined:

$$K_{ij}^{(e)} = t \cdot A \cdot B^T \cdot D \cdot B$$

And the forces:

$$f_b^{(e)} = \frac{A \cdot t^{(e)}}{3} \cdot \begin{bmatrix} 0 \\ -\rho g \end{bmatrix}$$

$$f_\varepsilon^{(e)} = \frac{t^{(e)}}{2} \cdot \begin{Bmatrix} b_i\left(d_{11}\varepsilon_x^0 + d_{12}\varepsilon_y^0\right) + c_i d_{33}\gamma_{xy}^0 \\ c_i\left(d_{21}\varepsilon\varepsilon\varepsilon_x^0 + d_{22}\varepsilon_y^0\right) + b_i d_{33}\gamma_{xy}^0 \end{Bmatrix}$$

And the system will have 12 degrees of freedom two for every node. The reduced system will have 4 DOF.

4. **Compute the FE approximation u^h. Use E=10GPa, v=0.2, $\delta=10^{-2}$m and $\rho g=10^3 N/mm^2$**

To solve the problem we reduce the system using the Boundary conditions described in the first question, and we get the next values:

$$a = \begin{bmatrix} a_7 = -0.0012 \\ a_8 = -0.0001 \\ a_{10} = -0.0005 \\ a_{12} = -0.0016 \end{bmatrix} m, \text{ and the reactions } r = \begin{bmatrix} r_1 = 13520000 \\ r_2 = 25021225 \\ r_3 = 72855000 \\ r_4 = -4213775 \\ r_5 = -5200000 \\ r_6 = 26001225 \\ r_9 = 57225000 \end{bmatrix} \text{N.}$$