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For solving the ODE we implement different methods as Euler, Heun and Runge-Kutta 4. These methods divide the interest interval [x_0, x_f] into n parts of length h,

$$h = \frac{x_f - x_0}{n}$$

Euler Method

The Euler method is the simplest of numerical methods to solve an initial value problem. It is a first-order method and comes from the Taylor's polynomial. The method can be expressed as

$$y_{i+1} = y_i + hf(x_i, y_i)$$

Heun Method (second order Runge-Kutta method)

Runge-Kutta methods replace the initial value problem with the equivalent integral equation.

The Heun methods applies the method of trapezoids to estimate the next unknown step. This can be finally expressed as

$$y_{n+1} = y_n + \frac{h}{2}\left[f\left(x_n, y_n\right) + f\left(x_{n+1}, y_{n+1}{}^*\right)\right]$$

which can also be presented the following way:

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{2}(k_1 + k_2) \\ k_1 &= f(x_n, y_n) \\ k_2 &= f(x_{n+1}, y_n + k_1) \end{aligned}$$

Runge-Kutta 4 (RK4)

Follows the same idea as the Heun method but integrating by the Simpson Method. The most usual fourth order method is that determined by the following formulas,

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\ k_3 &= f(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \\ k_4 &= f(x_n + h, y_n + k_3) \end{aligned}$$

Applying these methods to the proposed problem we obtain the following results that are compared with the analytical result (Figure 1). As we can observe the most accurate method is RK4 as it manages to obtain the same result as the analytic solution, but we have to take into account that this is the one that requires the greatest computational cost.

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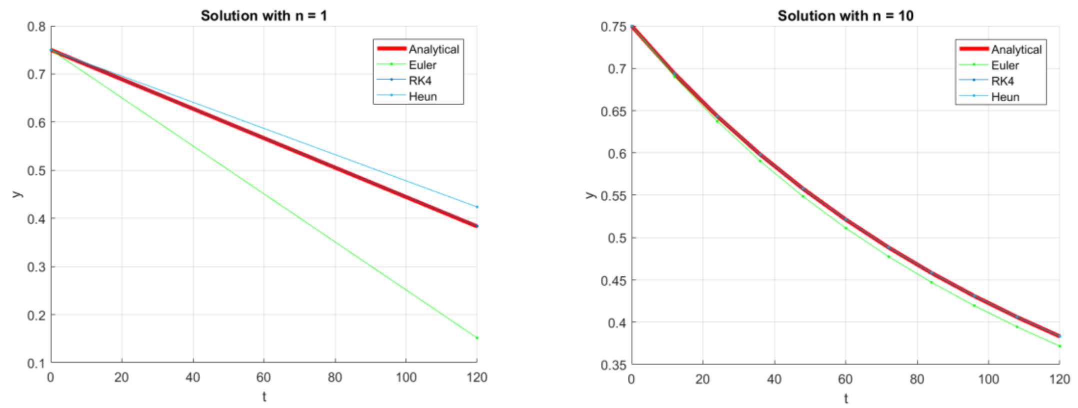

Figure 1: *Comparison of the results for different n*

Next, we calculate the error versus time and versus number of steps knowing that $y(120)$ = 0.383 m.

a) b)

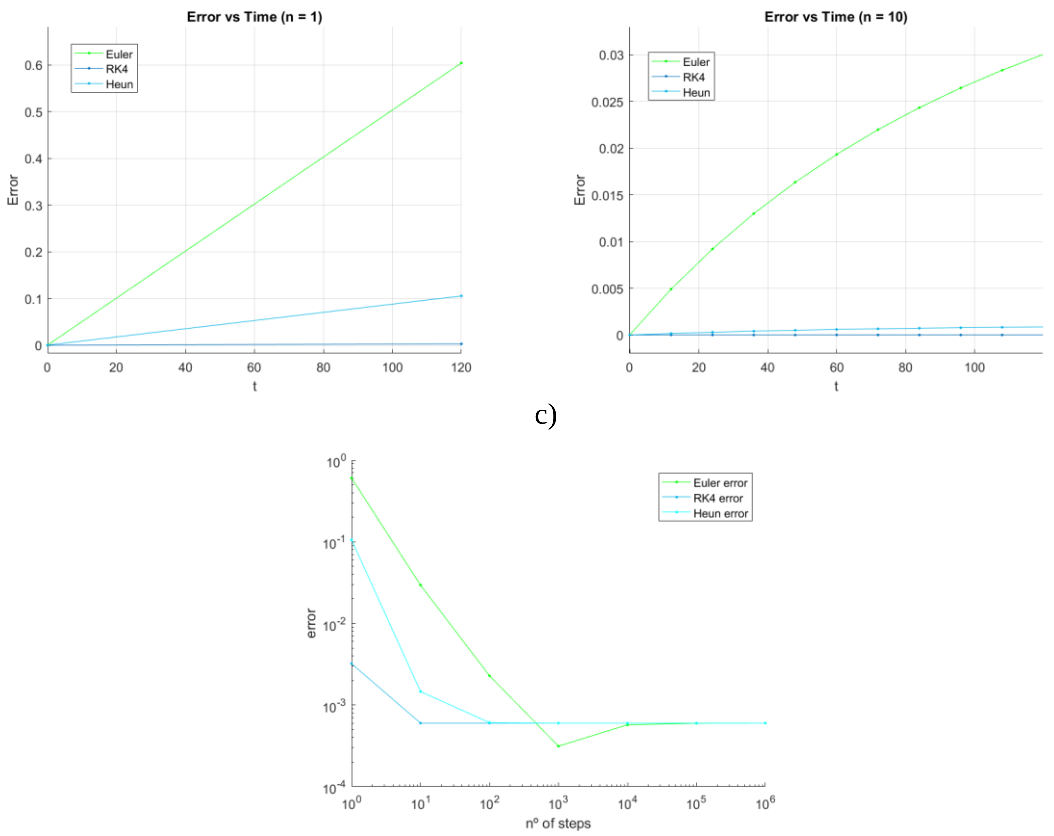

Figure 2: *error vs (a) time (n=1), (b) time (n=10) and (c) number of steps*

As we can observe in the

Figure 2 (b) as we increase the number of steps, the error decreases, except with the Euler method which reach a minimum value and then increases the error again. In the case of (a) and (b) we see that the error increases as the time does, which can be explained by the fact that with every step we have a sum of errors from previous steps. This is independent of the number of steps in the sense that always tends to grow with time, but we can observe that the magnitude of

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the error decreases for bigger n, simply because the error accumulated on previous steps is smaller.