PDE

$g = 9.8 \frac{m}{s^2}$

$L = 1m$

$h = -0.5$

1.) a.) 2-steps $\quad \frac{\partial^2\theta}{\partial t^2} + \frac{g}{L} \cdot \theta = 0$

Boundary

$\theta(1) = 0.4$ rad, position.

$\frac{d\theta}{dt}(1) = 0 \frac{rad}{s}$, velocity.

- $\frac{\partial\theta}{\partial t} = z$ (1) $\quad , \quad \theta(1) = 0.4$ rad.

  $f_1(\theta, t, z)$

$\frac{\partial z}{\partial t} + \frac{g}{L} \cdot \theta = 0$

- $\frac{\partial z}{\partial t} = \frac{-g}{L} \cdot \theta$ (2) $\quad , \quad \frac{\partial\theta}{\partial t}(1) = 0 \frac{rad}{s} \longrightarrow z(1) = 0 \frac{rad}{s}$.

  $f_2(\theta, t, z)$

- $y_{i+1} = y_i + \left(\frac{1}{2} K_1^y + \frac{1}{2} K_2^y\right) \cdot h$.
- $z_{i+1} = z_i + \left(\frac{1}{2} K_1^z + \frac{1}{2} K_2^z\right) h$.

a) $\cdot\ K_1^y = f_1(t_i, \theta_i, z_i)$

c) $\cdot\ K_2^y = f_1(t_i + h, \theta_i + K_1^y \cdot h, z_i + K_1^z \cdot h)$

b) $\cdot\ K_1^z = f_2(t_i, \theta_i, z_i)$

d.) $\cdot\ K_2^z = f_2(t_i + h, \theta_i + K_1^y h, z_i + K_1^z \cdot h)$

$\underline{i=0}$ $t_0 = 1$ $\theta_0 = 0.4\,rad$ $z_0 = 0\frac{rad}{s}$ $h = -0.5$ (2)

a.) $\boxed{K_1^y \rightarrow z = 0}$

b.) $K_1^z \rightarrow -\frac{g}{L}\cdot\theta \rightarrow -\frac{9.8\,m/s^2}{1\,m}\cdot 0.4\,rad \rightarrow \boxed{-3.92\frac{rad}{s^2} = K_1^z}$

c.) $K_2^y \rightarrow f_1(t_i + h, \theta_i + K_1^y \cdot h, z_i + K_1^z \cdot h)$

$f_1(1 + (-0.5), 0.4\,rad + (0\cdot(-0.5)), 0 + (-3.92)\cdot(-0.5))$

$f_1(0.5, 0.4\,rad, 1.96)$

$\boxed{K_2^y \rightarrow z = 1.96}$

d.) $K_2^z \rightarrow f_2(0.5, 0.4, 1.96)$

$K_2^z \rightarrow \frac{-9.8\,m/s^2}{1\,m}\cdot 0.4\,rad$

$\boxed{K_2^z = -3.92}$

$y_1 = y_0 + \frac{h}{2}(K_1^y + K_2^y)$

$y_1 = 0.4 - \frac{0.5}{2}(0 + 1.96)$

$\boxed{y_1 = -0.09\,rad}$

$z_1 = z_0 + \frac{h}{2}(K_1^z + K_2^z)$

$z_1 = 0 - \frac{0.5}{2}(-3.92 - 3.92)$

$\boxed{z_1 = 1.96\frac{rad}{s}}$

$i=1.$ $i=1$, $t_1 = 0.5\,s$, $\theta_1 = -0.09\,rad$, $z_1 = 1.96\,rad/s$, $h = -0.5\,s$.

(3)

• Calculate → $K_1^y, K_1^z \rightarrow K_2^y, K_2^z \rightarrow y_2, z_2$.

a.) $K_1^y = f_1(t_1, \theta_1, z_1)$

$\boxed{K_1^y \rightarrow z_1 = 1.96 \frac{rad}{s}}$

b.) $K_1^z = f_2(t_1, \theta_1, z_1)$

$K_1^z = f_2(0.5, -0.09, 1.96)$

$K_1^z = \frac{-9.8 \frac{m}{s^2}}{1m} \cdot -0.09\,rad \rightarrow \boxed{K_1^z = 0.882\,rad/s^2}$

c.) $K_2^y = f_1(t_1+h, \theta_1 + K_1^y \cdot h, z_1 + K_1^z \cdot h)$

$K_2^y = f_1\left(0.5s + (-0.5s), -0.09\,rad + \left(1.96 \frac{rad}{s} \cdot (-0.5s)\right), 1.96 \frac{rad}{s} + \left(0.882 \frac{rad}{s^2} \cdot (-0.5s)\right)\right)$

$K_2^y = f_1\left(0s, -1.07\,rad, 1.519 \frac{rad}{s}\right)$

$\boxed{K_2^y \rightarrow z_1 = 1.519 \frac{rad}{s}}$

d.) $K_2^z = f_2(t_1+h, \theta_1 + K_1^y \cdot h, z_1 + K_1^z \cdot h)$

$K_2^z = f_2\left(0s, -1.07\,rad, 1.519 \frac{rad}{s}\right)$

$K_2^z \rightarrow \frac{-9.8 \frac{m}{s^2}}{1m} \cdot -1.07\,rad \rightarrow \boxed{K_2^z = 10.486 \frac{rad}{s^2}}$

$y_2 = y_1 + h \cdot \left(\frac{K_1^y}{2} + \frac{K_2^y}{2}\right)$

$y_2 = -0.09\,rad + (-0.5s) \cdot \left(\frac{1.96}{2} \frac{rad}{s} + \frac{1.519}{2} \frac{rad}{s}\right) \rightarrow -0.09\,rad - 0.86975\,rad.$

$\boxed{y_2 = -0.95975\,rad}$

$z_2 = z_1 + h \cdot \left(\frac{K_1^z}{2} + \frac{K_2^z}{2}\right)$

$z_2 = 1.96 \frac{rad}{s} + (-0.5s) \cdot \left(\frac{0.882}{2} \frac{rad}{s^2} + \frac{10.486}{2} \frac{rad}{s^2}\right) \rightarrow 1.96 \frac{rad}{s} - 2.842 \frac{rad}{s}.$

$\boxed{z_2 = -0.882\,rad/s}$

4-steps RK 2nd order

* Same process, with $h = -0.25$.

i	y	z
0	0.4	0
1	0.278	0.981
2	-0.0530	1.361
3	-0.3768	0.8134
4	-0.4647	-0.3576

b.) Error

$$\frac{\theta_0^{h=0.5} - \theta_0^{h=0.25}}{\theta_0^{h=0.5}} = 0.5154$$

c.) Error 3x magnitude smaller.

New step size.

local error $\rightarrow O(h^2)$.

$$h_{prop} = \sqrt[3]{\left(\frac{\text{Tolerance}}{\text{Error}}\right)} \times h \rightarrow \left(\frac{\text{Error} \times 10^{-3}}{\text{Error}}\right)^{1/3} \times h$$

$h_{prop} = h \cdot \frac{1}{10}$ $\rightarrow$ h proposed should be $\frac{1}{10}$ of used h, to achieve smaller magnitude error.

2.) a.) $\frac{dy}{dx} = y - x^2 + 1$, $x \in (0,1)$, $y(0) = 1$.

$f(x,y) = y - x^2 + 1$

$y_{i+1} = y_i + f(x_i, y_i) \cdot h$

$i=0$

- $y_1 = y_0 + f(x_0, y_0) \cdot h$
  $= 1 + f(0,1) \cdot 0.25$
  $= 1 + [1 - (0)^2 + 1] \cdot 0.25$
  $= 1 + 0.5$
  $\boxed{y_1 = 1.5}$

$x_0 = 0$
$y(0) = y_0 = 1$
$h = 0.25$

- $y_2 = 1.5 + f(0.25, 1.5) \cdot 0.25$
  $= 1.5 + [1.5 - (0.25)^2 + 1] \cdot 0.25$
  $= 1.5 + 0.609375$
  $\boxed{y_2 = 2.109375}$

$x_1 = 0.25$
$y_1 = 1.5$

- $y_3 = 2.109375 + [2.109375 - (0.50)^2 + 1] \cdot 0.25$
  $\boxed{y_3 = 2.82421875}$

$x_2 = 0.50$
$y_2 = 2.109375$

- $y_4 = 2.82421875 + [2.82421875 - (0.75)^2 + 1] \cdot 0.25$
  $\boxed{y_4 = 3.639648438}$

$x_3 = 0.75$
$y_3 = 2.82421875$

$x_4 = 1.00$
$y_4 = 3.639648438$

1.) b.) For the Heun method to cost the same, since it evalutes twice as many variables, it should take half as many steps, therefore:

$h_{new} = 0.5$

i	y
0	1.0
1	2.1875
2	3.8361.

c.) Don't do.

d.) " ".

4.) " ".