Numerical methods for PDEs

Ordinary differential equalations

Exercise 2.

Using Euler method:

```
clear all; clc; close all;

f=@(x,y)y-x^2+1;
h=input('h = ');
x=[0:h:1];
y=zeros(1,length(x));
y(1)=input('y(0) = ');

for i=1:(length(x)-1)
    y(i+1)=y(i)+h*f(x(i),y(i));
end
```

y =

1.0000 1.5000 2.1094 2.8242 3.6396

Using Heun method:

```
clear all; clc; close all;

f=@(x,y)y-x^2+1;
h=input('h = ');
x=[0:h:1];
yy=zeros(1,length(x));
y=zeros(1,length(x));
y(1)=input('y(0) = ');
yy(1)=y(1);

for i=1:(length(x)-1)
    yy(i+1)=y(i)+h*f(x(i),y(i));
    y(i+1)=y(i)+(h/2)*(f(x(i),y(i))+f(x(i),yy(i+1)));
end
```

yy =

1.0000 1.5000 2.1875 3.0195 4.0016

Pure interpolation polynomial:

```
x1=[0 0.5 1];
el=ones(1,3)';
ms=x1';
hm=ms.^2;
f1=zeros(1,3)';
f1(1)=f(x1(1),yy(1));
f1(2)=f(x1(2),yy(3));
f1(3)=f(x1(3),yy(5));
M=[el ms hm];
a=zeros(3,1)';
a=M\f1;
```

$p(x)=2+1.7484*x+0.2532*x^2$

Exercise 3

$$f_{i+1} = f_i + \Delta x \left.\frac{df}{dx}\right|_i + \frac{\Delta x^2}{2} \left.\frac{d^2f}{dx^2}\right|_i$$

$$\left.\frac{df}{dx}\right|_i = \frac{f_{i+1} - f_i}{\Delta x} - \underbrace{\frac{\Delta x^2}{2}\frac{d^2f}{dx}}_{T_i^n(f,x)}$$

A method is consistent when $\Delta x \to 0$

Ex. 3 /

c) forward Euler-method:

$f(x,y) = \lambda y$

$y_{i+1} = y_i + h\lambda y_i = G_i \rightarrow G = 1 + \lambda h$

here: $\frac{dy}{dx} = -\lambda y$

$\Downarrow$

$\lambda = -1$

h = real postive

abs. stable:

$|1 + \lambda h| < 1 \rightarrow |1 + (-1)\cdot h| \overset{?}{<} 1$

$|1 - h| < 1$ ✓

YES, it is stable

d) backward Euler:

$\frac{dy}{dx} = f(x,y)$

$y_0 = 1$

$y_{i+1} = y_i + h \cdot f(x_{i+1}, y_{i+1})$

$y_i = y_{i+1} - h \frac{dy}{dx}(x_{i+1}) + O(h^2)$

$\frac{dy}{dx}(x_{i+1}) = \frac{y_{i+1} - y_i}{h} + \tau_i(h)$

$y_{i+1} = y_i + h f(x_{i+1}, y_{i+1}) + \underbrace{h \cdot \tau_i(h)}_{\text{neglect it for discretizing}}$

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d) $\frac{dy}{dx} = -25y^{3,5}$ $y(0)=1$

$h=0,1$

$y_0 = 1$

$y_1 = y_0 + h \cdot f(x_1, y_1)$

$y_1^* = y_0 - \frac{f}{f'} = y_0 - 0{,}2857 \cdot y_0 = (1-0{,}2857) \cdot y_0 = 0{,}7143 y_0$

$y_1^{**} = 0{,}7143 \cdot y_1^* = 0{,}5102$

$y_1 = 1 + 0{,}1 \cdot (-25 \cdot 0{,}5102^{3,5}) = 0{,}7628$

$y_2 = y_1 + h \cdot f(x_2, y_2) =$

$y_2^* = 0{,}7143 \cdot y_1 = 0{,}5449$

$y_2^{**} = 0{,}7143 \cdot y_2^* = 0{,}3892$

$y_2 = 0{,}7628 + 0{,}1\,(-25 \cdot 0{,}3892^{3,5}) = 0{,}6708$

e) $y_{i+1} = y_i + h \cdot f(x_i, y_i)$

$y_0 = 1$

$y_1 = y_0 + h \cdot f(x_0, y_0) = 1 + 0{,}1\,(-2{,}5 \cdot 1^{3,5}) = -1{,}5$

$y_2 = y_1 + h \cdot f(x_1, y_1) = -1{,}5 + 0{,}1\,(-2{,}5 \cdot (-1{,}5)^{3,5}) =$

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