Eszter DUDÁS

Numerical methods for PDEs

Exercises 3

I am really sorry about the format but the scanner doesn't work anymore so I have to make the photos with my mobile….

Exercise 2)

(2) $u_t + a u_x = 0$

a) $$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\,\frac{u_{i+1}^{n+1} - u_i^{n+1}}{\Delta x} = 0$$

$$u_i^{n+1} - u_i^n + \frac{a \cdot \Delta t}{\Delta x}\left[u_{i+1}^{n+1} - u_i^{n+1}\right] = 0$$

$$\left.\frac{du}{dx}\right|_i^n = \frac{u_{i+1}^n - u_i^n}{\Delta x} + \underbrace{\frac{1}{2}\Delta x \cdot \frac{d^2u}{dx^2}}_{\tau_i^n}$$

b)

$$u_i^{n+1} + \frac{a\Delta t}{\Delta x} \cdot u_{i+1}^{n+1} - \frac{a\Delta t}{\Delta x} u_i^{n+1} = u_i^n + \tau_i^{n+1}$$

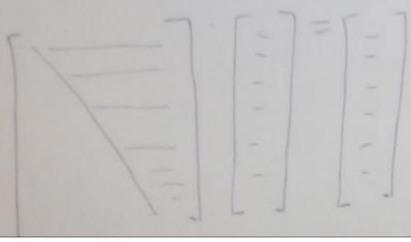
$$\left(1 - \frac{a\Delta t}{\Delta x}\right) \cdot u_i^{n+1} + \frac{a\Delta t}{\Delta x} \cdot u_{i+1}^{n+1} = u_i^n + \tau_i^{n+1}$$

$$\begin{bmatrix} \left(1 - \frac{a\Delta t}{\Delta x}\right) & \frac{a\Delta t}{\Delta x} & 0 \\ 0 & (\quad) & \frac{a\Delta t}{\Delta x} \\ 0 & 0 & (\quad) \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{m+1} \end{bmatrix}^{n+1} = \left\{ 1 \cdot u_i^n + \begin{bmatrix} \sin(2\pi \cdot 0) \\ 0 \\ 0 \\ 0 \\ \sin(2\pi \cdot 1) \end{bmatrix} \right.$$

c) direct method: Gauss-elimination

iterative method: Jacobi

d)



Exercise 4)

4) $u_t = \nu u_{xx} + \sigma u$

a) $$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \nu \left[\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} \right] + \sigma u_i^n$$

$$\left.\frac{du}{dx}\right|_{m+1}^{n} = \frac{u_{m+2}^n - u_m^n}{2\Delta x} + \mathcal{O}(\Delta x^2)$$

$$u_{m+2}^n = \left.\frac{du}{dx}\right|_{m+1}^{n} \cdot 2\Delta x + u_m^n$$

b) $\sigma = 0$: Forward in time centered in space sheme

$\nu = 0$: ordinary differential equation c)

$$u_i^{n+1} - u_i^n = \frac{\nu \cdot \Delta t}{\Delta x}\left[u_{i+1}^n - 2u_i^n + u_{i-1}^n\right] + G u_i^n \cdot \Delta t$$

$$u_i^{n+1} = \frac{\nu \Delta t}{\Delta x} \cdot u_{i+1}^n + \left(1 - \frac{2 \cdot \nu \cdot \Delta t}{\Delta x}\right) \cdot u_i^n + \frac{\nu \cdot \Delta t}{\Delta x} \cdot u_{i-1}^n + G \cdot \Delta t \cdot u_i$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^{n+1} = \begin{bmatrix} \left(1 - \frac{2\nu\Delta t}{\Delta x} + G\Delta t\right) & \frac{\nu\Delta t}{\Delta x} & 0 \\ \frac{\nu\Delta t}{\Delta x} & (\quad) & \frac{\nu\Delta t}{\Delta x} \\ 0 & \frac{\nu\Delta t}{\Delta x} & (\quad) \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^{n} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}^{n+1} = \begin{bmatrix} \text{see matlab code} \end{bmatrix}$$

d) implicit method

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \nu\left[\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{\Delta x^2}\right] + G u_i^{n+1} + \eta_i^{n+1}$$

$$\left.\frac{du}{dx}\right|_{m+1} = \frac{u_{mx+2} - u_{mx}}{2\Delta x} = 0$$

$$u_{mx+2} = u_{mx}$$

$$A = [\ \ldots\ ] \cdot \underset{U^{n+1}}{\begin{bmatrix} - \\ \vdots \\ - \end{bmatrix}} = \underset{u^n}{\begin{bmatrix} - \\ \vdots \\ - \end{bmatrix}} + \underset{F}{\begin{bmatrix} u_0^n \\ 0 \\ 0 \\ u_{mx+1}^n \end{bmatrix}}$$

Most suitable method is Gauss-Seidel to solve it

Explicit method:

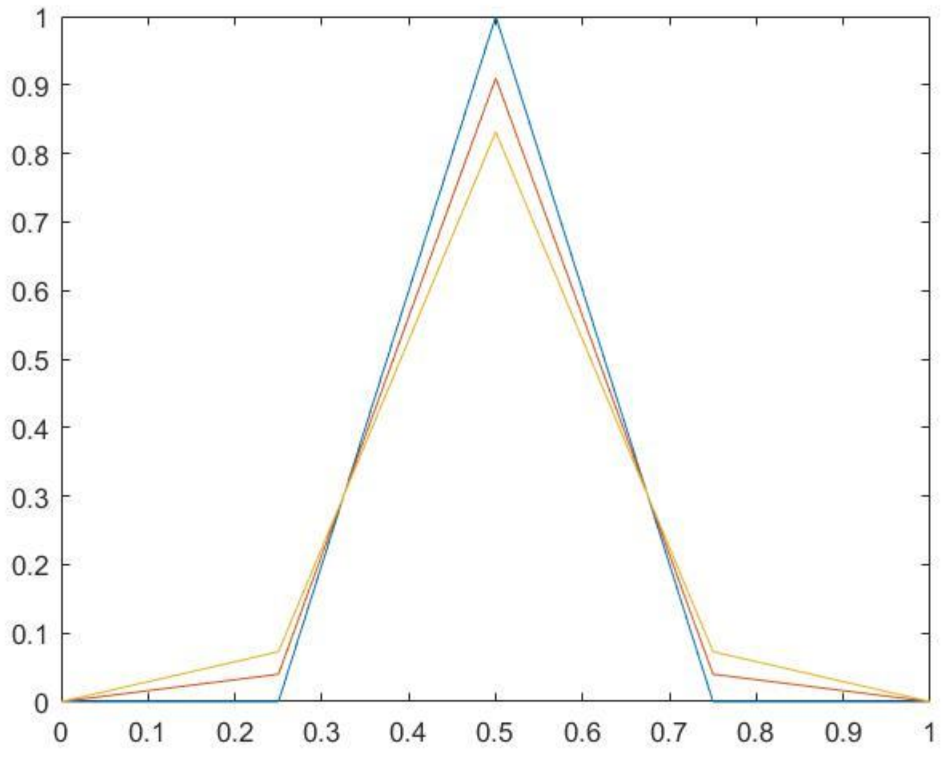
```
v=0.1;
o=-0.1;
dx=0.25;
dt=0.1;

a=1-(2*v*dt)/(dx)+o*dt;
b=(v*dt)/dx;

A=[a b 0; b a b; 0 b a];
U=[0 1 0]';

U1=A*U;

U2=A*U1
```



U1 =[0.0400; 0.9100;0.0400]

U2 =[0.0728;0.8313;0.0728]