AZELART HENRI

Numerical Methods for Partial Differential Equations.

1/ Initialize

$k = 0$

$x^0 = \sqrt[3]{20}$

$f(x) = x^3 + 2x^2 + 10x - 20$

$f'(x) = 3x + 4x + 10$

1 Iteration

$f(x^0) = 41,880$

$f'(x^0) = 42,962$

$\Rightarrow x^1 = x^0 - \frac{f(x^0)}{f'(x^0)} = 1,740$

2 Iteration

$f(x^1) = 8,223$

$f'(x^1) = 26,043$

$\Rightarrow x^2 = 1,405$

3 Iteration

$f(x^2) = 0,772$

$f'(x^2) = 21,542$

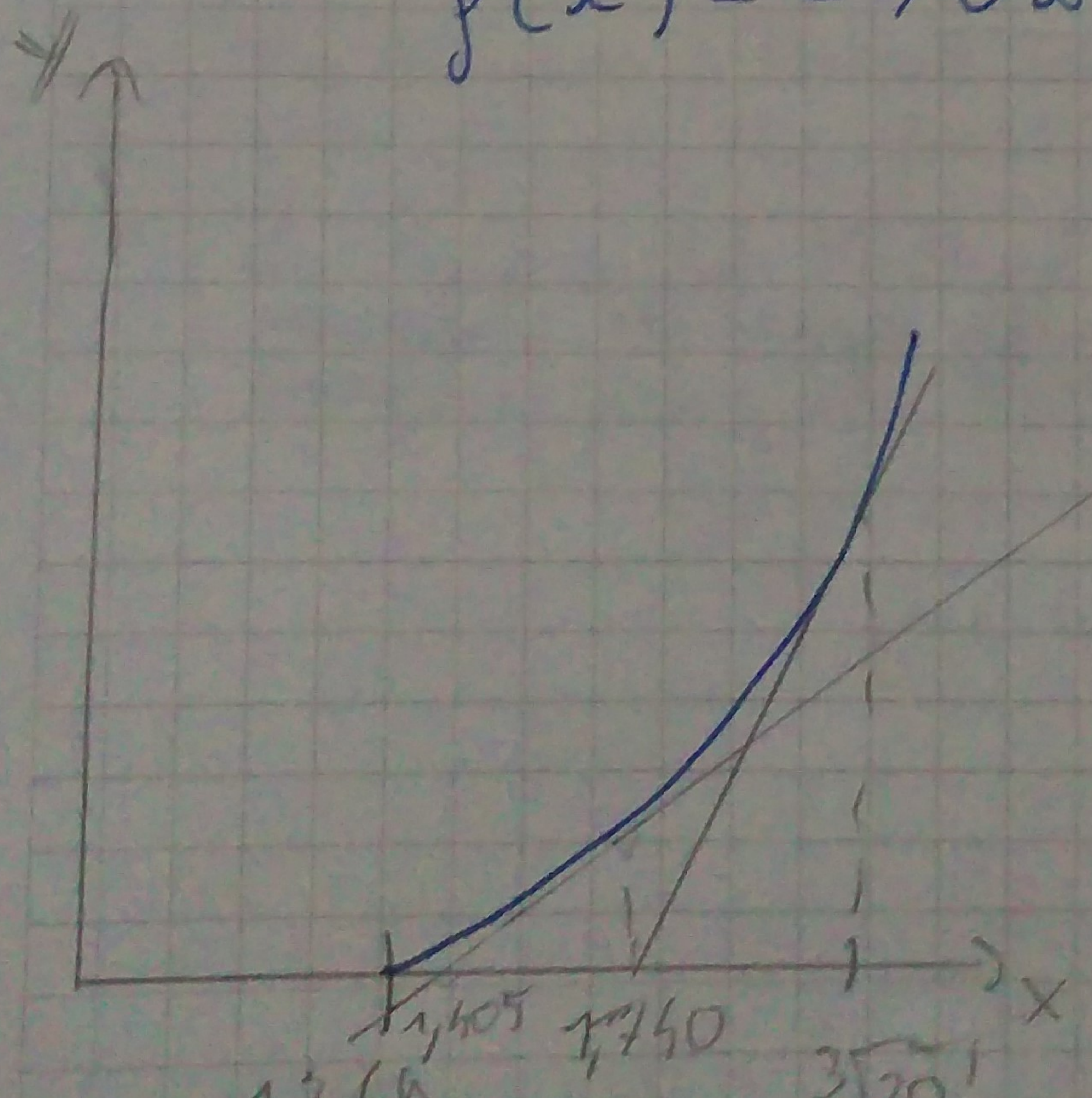
$\Rightarrow x^3 = 1,369$

4 Iteration

$f(x^3) = 0,004$

$f'(x^3) = 21,098$

$\Rightarrow x^4 = 1,369$



The Newton's Method Converge quickly to the root in only 4 iterations.

It behaves as expected.

5/ a) Integration and weights are chosen so polynomials of degree up to $2n+1$ are integrated exactly.

$2n+1=3 \Rightarrow n=1$

For $\int_{-1}^{1} f(z)\,dz \simeq \sum_{i=0}^{n} w_i f(z_i)$

$z_0 = -\frac{\sqrt{3}}{3} \quad z_1 = \frac{\sqrt{3}}{3}$

$w_0 = w_1 = 1$

$x_0 = \frac{b-a}{2} z_0 + \frac{a+b}{2}$ with $a=0$ $b=1$

$$x_0 = \frac{3-\sqrt{3}}{6} \qquad x_1 = \frac{3+\sqrt{3}}{6}$$

$$w_0^* = w_1^* = \frac{b-a}{2} w_0 = \frac{1}{2}$$

b) It's possible because $n=3$ with equally spaced points ($h=\frac{1}{4}$), the order is n.

$$w_0 = w_3 = \frac{3h}{8} = \frac{3}{32}$$

$$w_1 = w_2 = \frac{9h}{8} = \frac{9}{38}$$

6 a) Order $= (2[n+1]+1) = 2n+3$

b) $n=2 \Rightarrow$ Order $=5$

Polynomials of order exactly equal or inferior can be integrated exactly:

The answers are i) and ii)

AZELART Henri

7 i) . $\int_0^1 12x\,dx = \frac{h}{2}(f(0) + f(\frac{1}{2})) + \frac{h}{2}(f(\frac{1}{2}) + f(1)) = I$

with $h = \frac{1}{2}$

$\boxed{I = 6}$ $\quad E = -\frac{h^3}{6} f''(\mu) = 0$

. $\int_0^1 (5x^3 + 2x)\,dx = 0{,}25 . 1{,}625 + 0{,}25(1{,}625 + 7)$

$\boxed{I = 2{,}5625}$

$E = \frac{-h^3}{6} f''(1) = 0{,}625$

ii) . $\int_0^1 12x\,dx = \frac{h}{3}(f(0) + 4 . f(\frac{1}{2}) + f(1))$ with $h = \frac{1}{2}$

$= \frac{1}{6}(0 + 4.6 + 12)$

$\boxed{I = 6}$ $\quad E = -\frac{h^5}{90} f^{(4)}(\mu) = 0$

. $\int_0^1 (5x^3 + 2x)\,dx = \frac{1}{6}(0 + 4 . 1{,}625 + 7)$

$\boxed{I = 2{,}25}$

$E = -\frac{h^5}{90} f^{(4)}(\mu) = 0$

The Simpson's Method work as predicted with an error of 0.
Even if $n=2$, the function $5x^3+2x$ is exactly integrated.
The trapezoidal Method as an error pretty big as we can imagine using this Method with 2 intervals

10.

$$I = \int_0^1 \int_0^1 (9x^3 + 8x^2)(y^3 + y)\, dx\, dy$$

$$= \int_0^1 (y^3 + y)\, dy \int_0^1 (9x^3 + 8x^2)\, dx$$

$$\int_0^1 (9x^3 + 8x^2)\, dx = \frac{1}{6}\left(f(0) + 4f(\tfrac{1}{2}) + f(1)\right) \quad \text{with } h = \frac{1}{2}$$

$$= \frac{1}{6}(0 + 12{,}5 + 17)$$

$$= \frac{59}{12} = 4{,}917 \qquad E = 0$$

$$\int_0^1 (y^3 + y)\, dy = \frac{1}{6}(0 + 2{,}5 + 2)$$

$$= \frac{3}{4} = 0{,}75 \qquad E = 0$$

$$\Rightarrow \boxed{I = \frac{177}{48} = 3{,}6875}$$

The error behave like it would be a polynom of 3rd degree.