STRUCTURAL OPTIMIZATION

COMMUNICATION SKILLS II

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MSC IN NUMERICAL METHODS IN ENGINEERING

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MOTIVATION

OPTIMIZATION

Problem statement

$$\text{Find } \boldsymbol{x} \text{ to minimize } f(\boldsymbol{x}) \text{ subject to } \boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0}$$

f: objective function

x: n-vector

g: constraints, m-vector

$$\boldsymbol{g}(\boldsymbol{x}) \leq 0 \quad \Rightarrow \quad \begin{cases} g_1 \leq 0 \\ g_2 \leq 0 \\ \vdots \\ g_m \leq 0 \end{cases}$$

CONSTRAINTS

The constraints *g* can either satisfy:

- **An equality → Lagrange multipliers strategy**

$$\text{minimize } f(x) \mid g(x) = 0 \quad \Leftrightarrow \quad \nabla f + \lambda^T \nabla g = 0, \; g = 0.$$

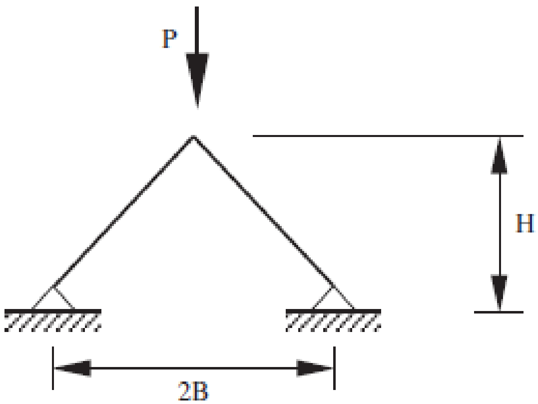
- **An inequality → Karush Kuhn Tucker conditions**

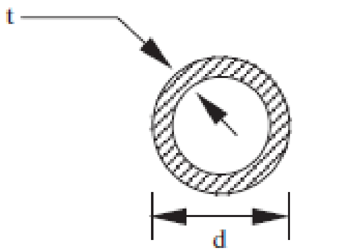
Theorem: (Kuhn–Tucker) At an optimal point of the problem minimize f(x) subject to $g(x) \leq 0$*, there exist Lagrange multipliers* $\lambda \geq 0$ *which satisfy* $\nabla f + \lambda^T \nabla g = 0$ *and* $\lambda_i g_i = 0$ *for* $i = 1,\dots,m$.

OPTIMIZATION

A simple example:

Parameter	Description	Value
E	Young's modulus	29,000 ksi
B	Half-distance between supports	100 in.
F_y	Yield stress of material	36 ksi
t	Wall thickness of tube	0.25 in.
P	Applied load	100 k





Member Cross Section

Item	Equation
Second moment of inertia (in.[4])	$I = \frac{\pi}{64}\left[(d+t)^4 - (d-t)^4\right] = \frac{\pi t d}{8}(d^2 + t^2)$
Member force (k)	$F = \frac{P}{2}\frac{\sqrt{B^2 + H^2}}{H}$
Member stress (ksi)	$\sigma = \frac{F}{A}$
Buckling stress (ksi)	$\sigma_{cr} = \frac{\pi^2 E I}{L^2}\frac{1}{A}$

$$\min f(\mathbf{x}) \text{ subject to } \mathbf{g}(\mathbf{x})$$

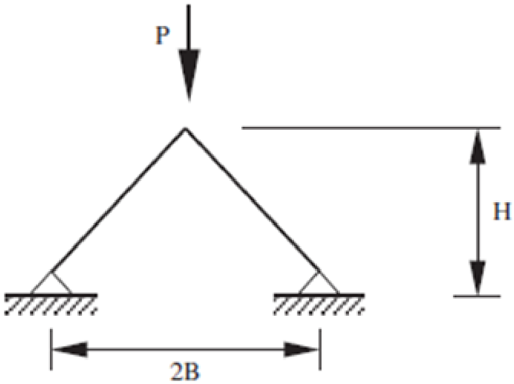
$$f = 2(d\,t\,\pi)\sqrt{H^2 + B^2}$$

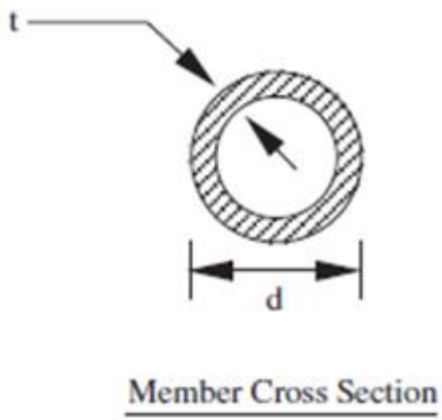
$$g_1 = \frac{P}{2}\frac{\sqrt{H^2 + B^2}}{H}\frac{1}{d\,t\,\pi} - F_y \le 0$$

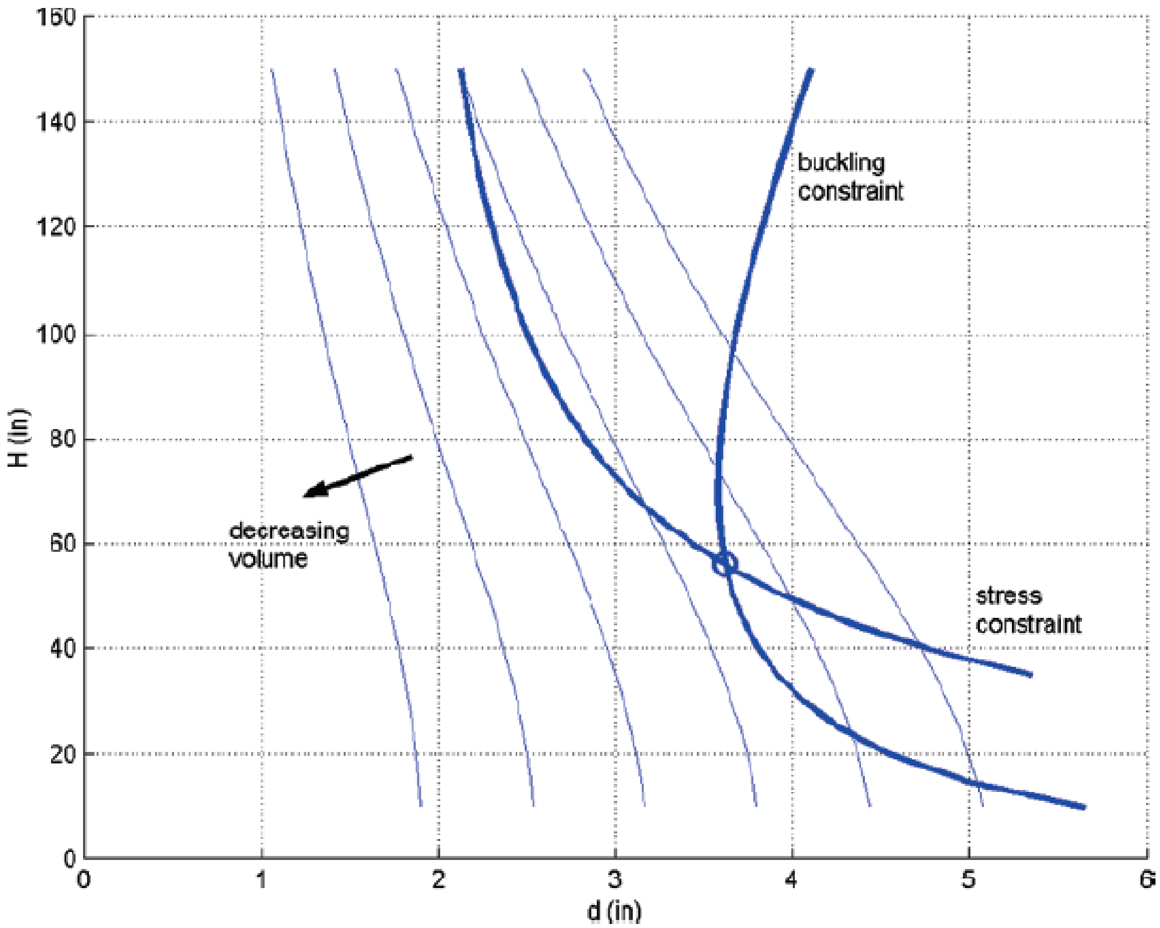
$$g_2 = \frac{P}{2}\frac{\sqrt{H^2 + B^2}}{H}\frac{1}{d\,t\,\pi} - \frac{\pi^2 E\left(d^2 + t^2\right)}{8\left(H^2 + B^2\right)} \le 0$$

OPTIMIZATION

A simple example:







LINEAR PROGRAMMING

- **Simplex method**

$$\text{Find } x \text{ to minimize } c^T x \quad \text{subject to} \quad Ax = b, \quad x \geq 0$$

- Partition of the matrix A

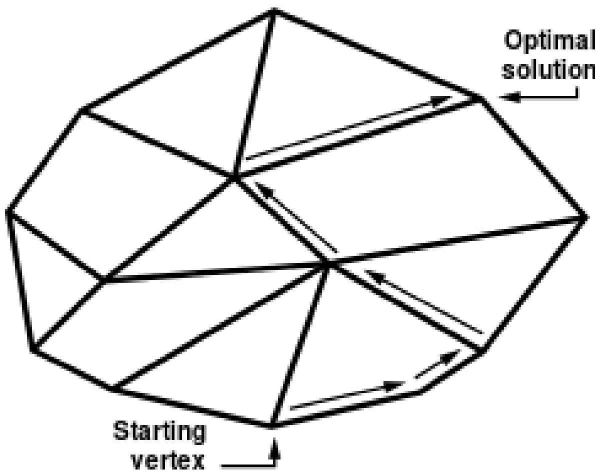
$$Ax = b \quad \Rightarrow \quad A_B x_B + A_N x_N = b \quad \Rightarrow \quad x_B = A_B^{-1}(b - A_N x_N)$$

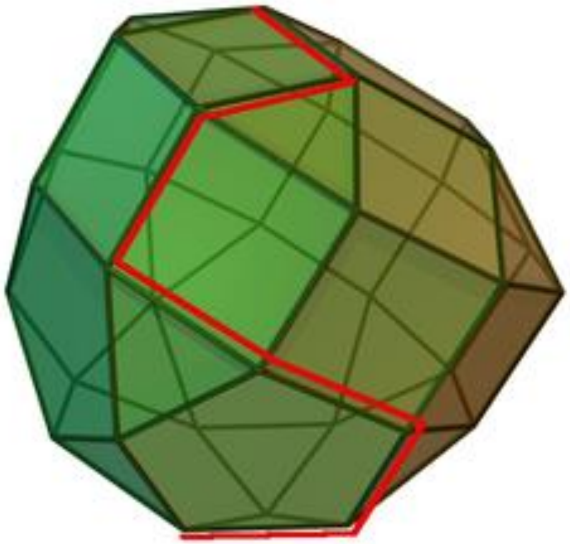
$$\begin{aligned} c^T x &= c_B^T x_B + c_N^T x_N = c_B^T A_B^{-1}(b - A_N x_N) + c_N^T x_N \\ &= c_B^T A_B^{-1} b + (c_N^T - c_B^T A_B^{-1} A_N) x_N \end{aligned}$$

- The simplex method starts with some set $x_N = 0$ and the coefficient of each $(x_N)_i$ is then examined in $c^T x = 0$
- If there is any negative coefficient, that variable increases to reduce f(x). The limit of increase is when some x_B is 0.
- By this process, there is one variable going into the basis and another one coming out

LINEAR PROGRAMMING

- **Simplex method**
  - The simplex method begins at a starting vertex and moves along the edges of the polytope until it reaches the vertex of the optimal solution





LINEAR PROGRAMMING

- **Interior point methods**
  - Moves along the interior points of the feasible domain

$$minimize \ c^T x \quad subject\ to\ Ax = b\ and\ x \geq 0$$

  - Looks for an incremental change dx such that

$$c^T x_{new} \leq c^T x_o \quad and \quad Ax_{new} = b$$

  - It turns out that dx is taken to be $-Pc$,

    with the projection $P = I - A^T(AA^T)^{-1}A$
  - Then

$$c^T dx = -c^T Pc = -c^T P^2 c = -\|Pc\| \leq 0$$

STRUCTURAL DESIGN

- Analysis tools, such as FE solvers
- Redesign
- Reanalyze

Clearly, the response of an elastic structure depends on its stiffness

LINEAR SYSTEMS VS OPTIMIZATION

It is frequently possible to replace a linear system by an optimization problem:

Given a set of linear equations

$$Ax = b$$

being the matrix A positive definite, an equivalent optimization problem is

$$\text{minimize } \Phi(x) = \tfrac{1}{2} x^T A x - x^T b$$

SEQUENTIAL LINEAR PROGRAMMING

Linear programming, together with the incremental equations of structures, provides a robust format form which to solve problems of structural optimization

Typically, a structural optimization problem is stated as find K, the matrix of stiffnesses, to satisfy the equations of structures, together with some displacement or stress constraints and minimize the structural volume or cost

The incremental version of this problem starts with some given solution and looks for a dK that satisfies certain constraints(*)

→ **Sequential linear programming problem**

(*) Examples of possible constraints: Move limits, scaling, regions of trust displacement constraints, stress constraints,…

OPTIMIZATION REVIEW

Nonlinear programming roots: Seminal paper by Lucien Schmidt, 1960's

1970's: Difficulties even for small optimization problems

1990's: Discussions of mathematical programming methods for solving large systems.

Actually, we can solve a nonlinear programming problem with thousands of millions of rows and columns.

COMPLEX OPTIMIZATION

Multiple loading conditions

Primal Problem

$$minimize\ \phi = \frac{\sigma}{E}\sum_i L_i \max\{|F_i^1|, |F_i^2|\}$$

$$subject\ to\ \ N^T F^1 = P^1\ and\ N^T F^2 = P^2$$

Dual Problem

$$maximize\ \psi = (P^1)^T \delta^1 + (P^2)^T \delta^2$$

$$subject\ to\ \ | N\delta^1 | + | N\delta^2 | \leq \frac{\sigma L}{E}$$

COMPLEX OPTIMIZATION

Multicriterion optimization

$$minimize\ f(x) = \left[f_1(x), f_2(x), \ldots, f_{n_1}(x) \right] \qquad subject\ to \qquad g(x) \le 0$$

- **A priori approach: The different targets are combined into a single one, turning the multi-objective problem into a single-objective one.**
- **Progressive approach: decision making and optimization are intertwined.**
- **A posteriori approach: a set of optimal candidate solutions (Pareto set) is obtained through the optimization process. Aftegr that, the most convenient solution is chosen.**

COMPLEX OPTIMIZATION

Incremental equations when shape change is allowed

	Equations of structures	Incremental form
Equilibrium equations	$N^T F = P$	$N^T dF + dN^T F = 0$
Constitutive equations	$F = K \Delta$	$dF = dK \Delta + K d\Delta$
Node/member displacement	$\Delta = N \delta$	$d\Delta = N d\delta + dN \delta$

The shape optimization problem, after some algebra, turns out to be:

$$\text{find } dF_i \text{ and } dR_i \text{ to}$$

$$minimize\ \varphi = \sum \left[(\operatorname{sgn} F_i) L_i dF_i + (N^T \mid F \mid)_i dR_i \right]$$

$$subject\ to \quad K_G dR + N^T dF = 0$$

COMPLEX OPTIMIZATION

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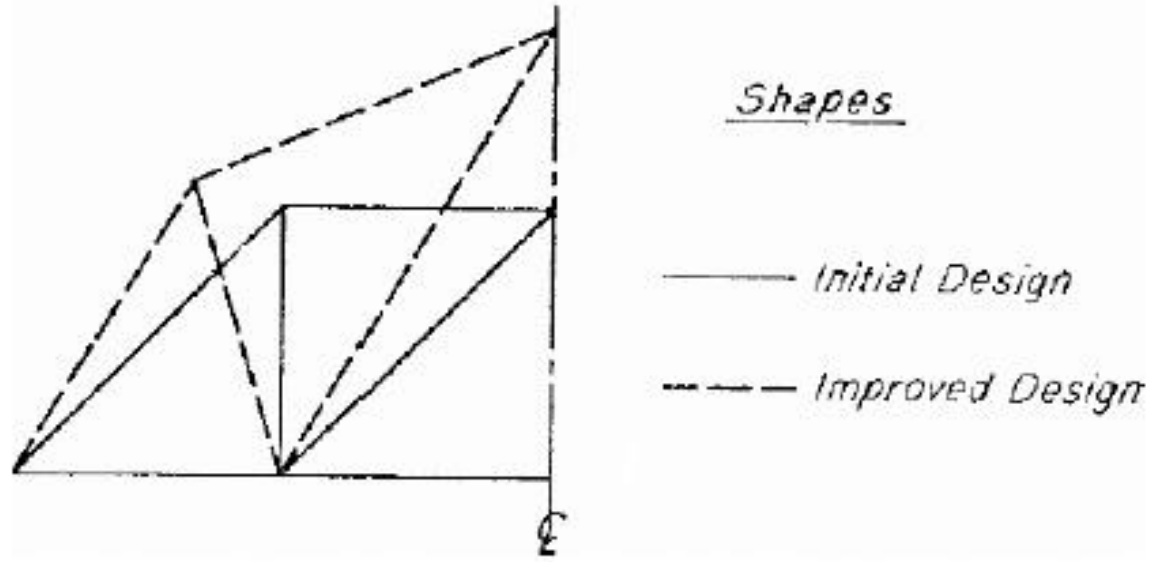
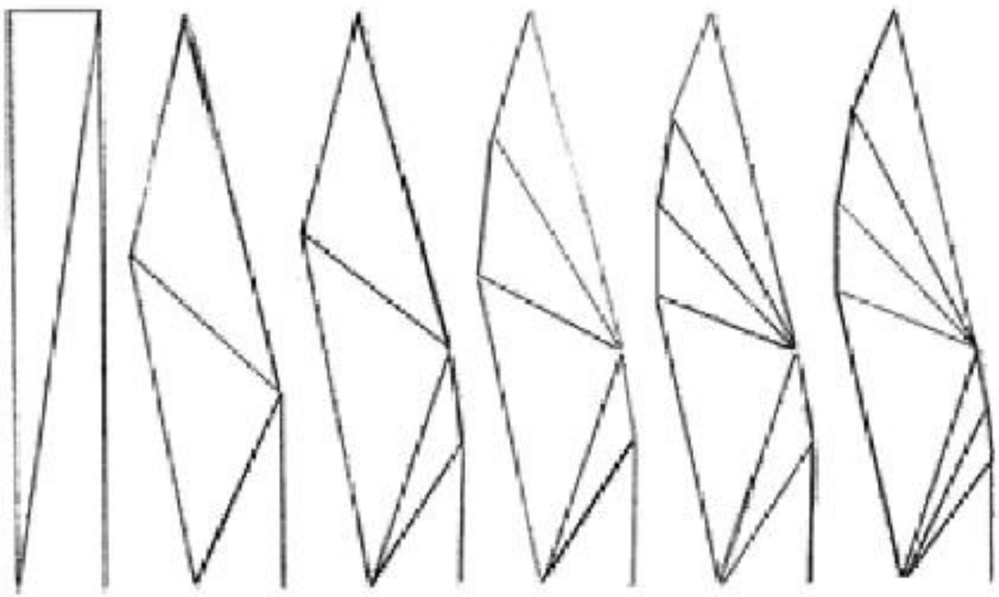


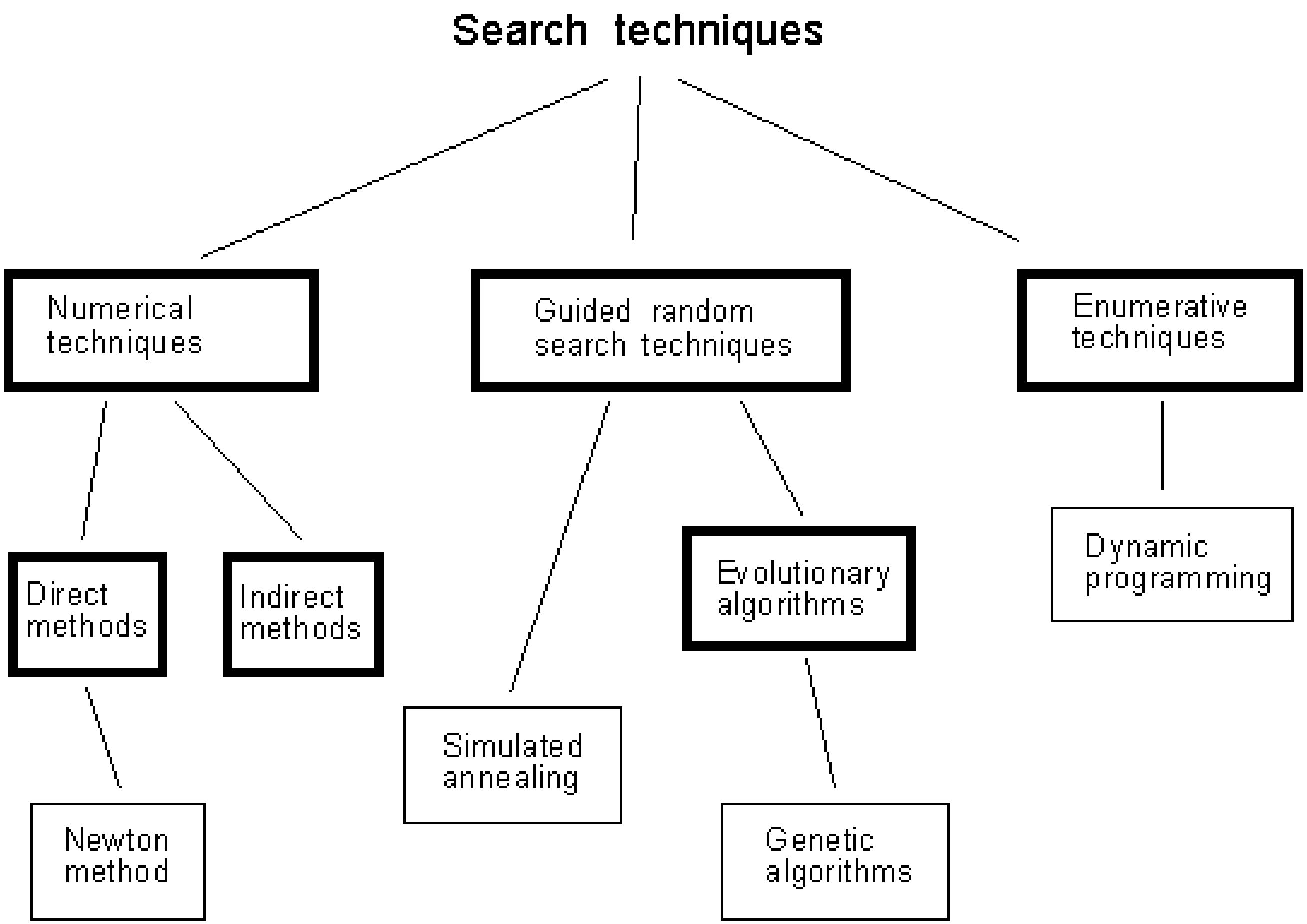
COMPLEX OPTIMIZATION

Generating new designs automatically



…an active area of research

AUTOMATED SHAPE OPTIMIZATION

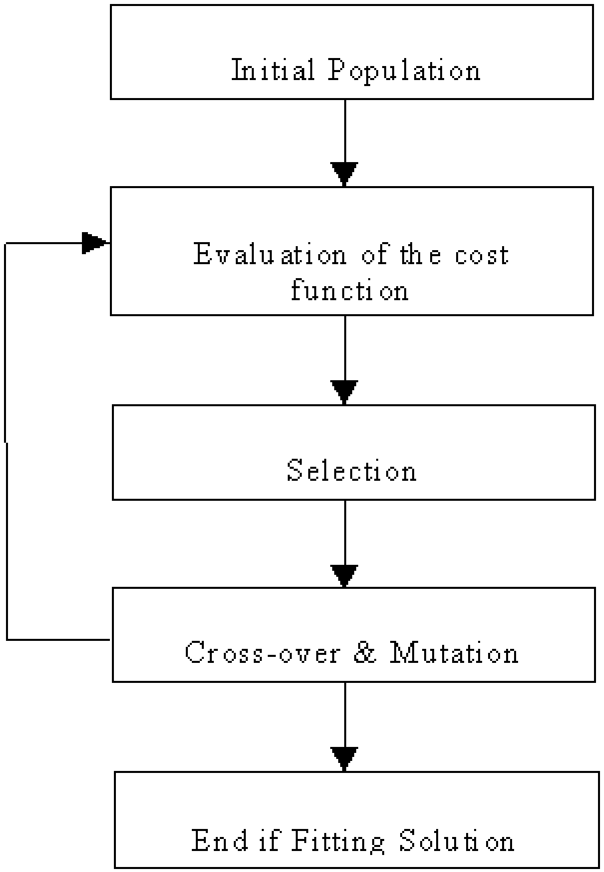


GENETIC ALGORITHMS

- **Heuristic algorithms**
- **They model the natural selection process of biology**

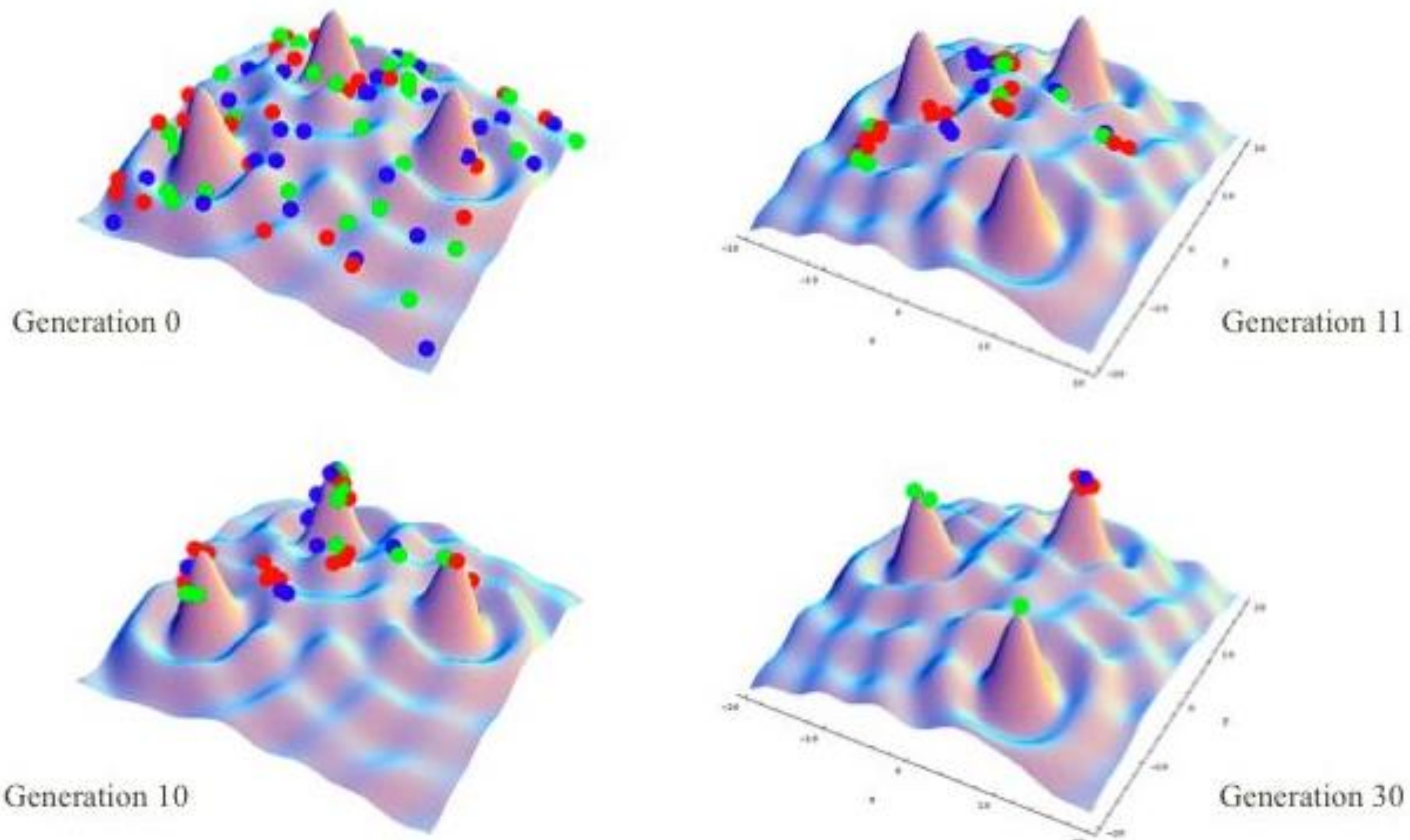
  - Crossover: two or more previous candidates are compared and parts are taken from each

  - Mutation: there is the potential for change within an individual design



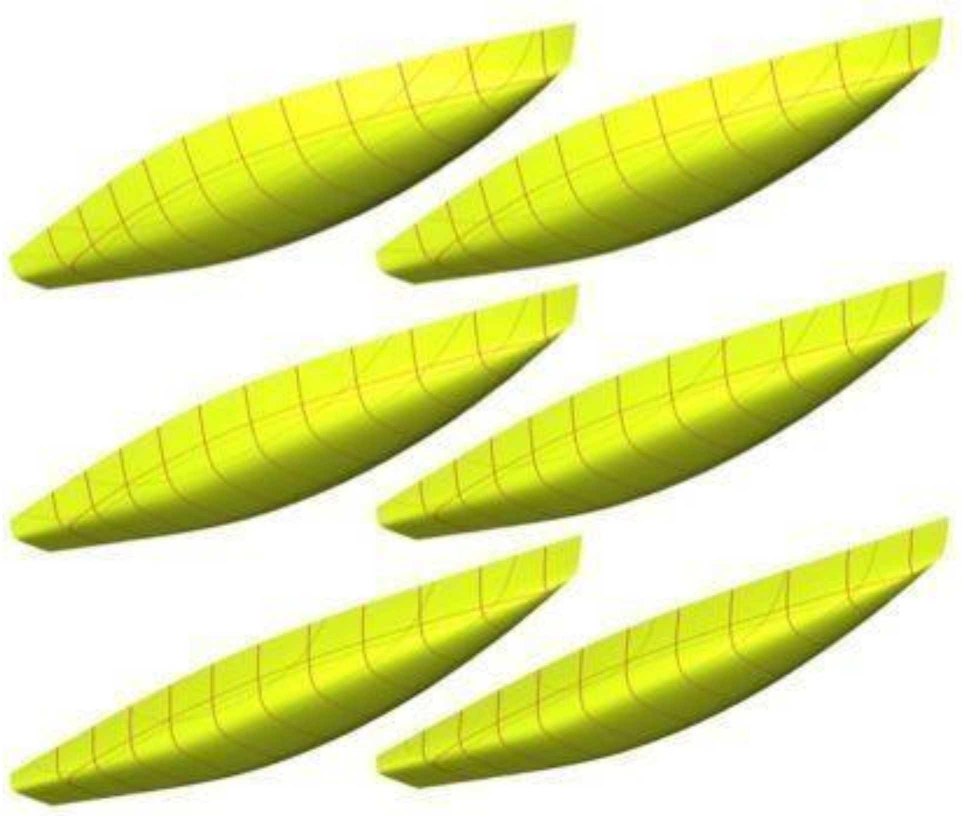
GENETIC ALGORITHMS

Evolution of the optimals while varying the objective function



GENETIC ALGORITHMS

Crossover and mutation given an initial set of shapes



CONCLUSIONS

We can think about mixing sequential linear programming together with the incremental equations of structures

→ **a general structural optimization solver**

- **Parametric description of the geometry, and the restrictions (control points, splines, …?)**
- **Finite Elements solver**
- **Constraints for the shape evolution**
- **Stress and strains constraints**

→ **Solve the optimization problem**