MATHEMATICS FOR AIRCRAFT DESIGN
Communication Skills II - Master Numerical Methods

Alba Navarro Casanova

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1 Introduction

Over the years we have seen a revolution in external design in different sectors, such as the aeronautical sector, the naval sector, and the automotive industry. Even this revolution has come to the world of sport. Elements that can be considered less sophisticated, such as the cyclists' helmets and the costumes worn by competition motorcyclists, have evolved to achieve an optimum aerodynamic design that improves the performance of the athlete and at the same time decreases their effort and resistance.

As an example we can observe how the cyclic helmets used in race against time, have been modified until arriving at the current design: now these are rounded in the frontal and angular in the occipital part. On the other hand, it can also be seen how the competition costumes of the motorcycle riders show curious humps, whose function is essentially aerodynamic, since it provides a later angled fairing to the helmet of the pilot without needing to alter the own form of the helmet, which would go to the detriment of security. These products named above have not undergone these changes for aesthetic issue, but for an aerodynamic issue, in order to optimize the performance of vehicles or athletes.

2 The optimization process

This process of optimizing the design is done by solving equations that allows the simulation of the behavior of a solid object (helmet or bicycle) in interaction with a fluid (eg air) to find the most efficient design, so the one that opposes less resistance to advance. In aircraft, vehicles or ships similar procedures are used, taking into account the peculiarities of each object and its environment. And on the other hand, depending on the final objective to be optimized, the so-called "objective function" will change. In this way we can optimize things like: speed, object stability or fuel expenditure.

3 The aeronautical sector

If we focus our attention on this sector we will be able to see how at world level they have set very ambitious goals with the aim of designing planes that respect the environment but in a more economic way. As an example, the European Union (EU) has set the medium-term goals ("Vision 2020") for the European aeronautics industry, among others:

• Strengthen competitiveness by reducing the costs of development and direct operation of aircraft and improving passenger comfort.

• Improve the environmental impact in relation to the emission of pollutants and the generation of noise.

For this the objectives to be achieved are the following:

• Reduce by 50% of CO2 emissions, which means a reduction of 50% in fuel consumption.

• Reduce by half the noise generated by aircraft.

• Reduce by half the time needed to introduce a new aircraft to the world market.

Although at first glance these three points seem to be unrelated, they all aim to refine the design of the aircraft in the aviation industry. An adequate design process not only affects the manufacturing costs of an aircraft, we have to keep in mind that the faster this process, the sooner the product can be offered to the market. But also that a reduction of the resistance of the aircraft directly translates into a decrease of the fuel consumption, that is to say, it also affects the operating costs.

In this sense, the key concept is the optimization or the so-called optimum design, that is the investigation of the external form of the aircraft that is aerodynamically more efficient, within certain structural constraints. For this reason, we can conclude that the decrease of the resistance to the advance of an airplane in the air is a key factor in the optimization of the shape of the surface of the aircraft.

In order to calculate this resistance to the advance we use the Navier-Stokes equations and, later and through optimization methods, the form of the airplane that minimizes the resistance is located, fulfilling other physical and geometric requirements.

4 The aeronautical design process

Nowadays practically all the optimization process is done by the use of the computer, reason why its degree of automation is very high.

Even so, the technology is not sufficiently developed to dispense with the obligatory and costly passage through the wind tunnel, which is used to validate the behavior of designs identified as promising through computer simulation. The origin of this design methodology dates back to the 1970s, at that time the use of Computational Fluid Dynamics (CFD) as a design tool began to be considered. Thanks in large part to the development of increasingly efficient numerical algorithms and the exponential growth of calculating capacity at reasonable prices CFD has become increasingly important.

Precisely the application of the mathematical theory of the control of systems governed by equations in partial derivatives has allowed to create the appropriate conditions for the formulation and resolution of problems of aerodynamic design with a computational cost assumable.

Usually the techniques of optimization for the design of aircraft use the denominated methods based on the gradients that can be systematized as:

1. The definition set of design variables that parametrize the external form of the aircraft.

2. The definition of an objective function or appropriate cost function. In practical terms it is usually a numerical parameter whose value is to be optimized. For this to choose properly the objective function is crucial because of it depends that an optimal configuration can be found. The most common objective functions are: the resistance to the advance of the aircraft, the support of the airplane and overall parameters of efficiency

3. The calculation of the gradient of the objective function with respect to the design variables. This gradient is used to determine the combination of variations of the design variables that give rise to an improvement in the value of the objective function and as a consequence the geometry is deformed. We are repeating this process until reaching a configuration that minimizes the objective function proposed.

To calculate the gradients in the most immediate way the direct derivation or finite difference method is used. This method consists of producing small disturbances in each and every one of the design variables and calculating the value of the objective function before and after each disturbance. The gradients can be calculated as the quotient between the variation of the objective function and the variation of the corresponding design variable. The disadvantage is that this procedure requires a numerical simulation of the air flow for each design variable, so that if their number is high (which is what normally happens), the computational cost is very high, we can assume that. Since each simulation, in complex but realistic problems, may require several days of computation in high-performance machines.

As a solution to the problems raised above appears the theory of control systems governed by equations in partial derivatives. This theory has been developed from the works of the french mathematician J. L Lions (among others) and is used for the indirect calculation of the gradients from the resolution of the so-called enclosed equation.

It is interesting because in this case, the computational cost required to obtain the objective function gradients in each design variable is negligible compared to the cost of a flow simulation, so that the cost is, for practical purposes, independent of the number of design variables, being equivalent to the cost of a flow simulation and a numerical resolution of the attached equation (linear equation, which means less complex).

The first time this methodology was applied was in the design of aerodynamic profiles (wings sections) in transonic regime (A. Jameson, professor of the Universities of Pricenton and Stanford, in 1988).

Over the years, considerable progress has been made in the application of these techniques to the design of complete aircraft, providing optimized wing designs, wing-fuselage configurations, tail surfaces, turbomachinery, and others.

Nowadays one of the current topics is to improve the efficiency of these design methods by searching for efficient numerical algorithms for the resolution of the attached equations, in a finer analysis of the optimization process in the presence of shock waves, in to deepen the knowledge and application of control theory to the calculation of objective function gradients of aeronautical interest. However, efforts are focused on the development of alternative mathematical optimization methods, such as the use of topological derivatives, level sets techniques, homogenization, multiobjective optimization and genetic algorithms.

5 For more information

Interesting webs:

- European Union web site on aeronautical research:
http://ec.europa.eu/research/aeronautics/index$_e n.html$

- M. Giles web:
http://web.comlab.ox.ac.uk/oucl/work/mike.giles/adjoint.html