

1D modelling of multi-frequency electromagnetic induction data

MARTA CODINA
Universitat de Barcelona
mcodinge7@ub.edu

Abstract

The electromagnetic induction methods are methods that allow to measure the electrical conductivity of the subsoil. There are some popular instruments for near-surface studies, as loop-loop instruments. Here I present a study of the conductivity of the subsoil using horizontal and coplanar loops (HCP), with an application of a data set collected over a field at three different frequencies: 1, 8 and 16 kHz. This instrument provides the so called apparent conductivity. To interpret the data it is necessary to solve the forward problem for different earth models: half-space, two and three layered models. By changing the parameters of the model to recover the experimental data of forward problem, one can estimate the value of the apparent conductivity. Taking into account a homogeneous medium, its conductivity value vary from 43.19 mS/m to 45.46 mS/m. In a two layered case, the values of the first medium vary between 45.46 mS/m and 50.00 mS/m. On the second medium, it varies from 10.00 to 25.00 mS/m. The width is 15-16 meters. And in a three layered case, the conductivity of the first medium is around 50.00 mS/m, the second and the last ones vary from 10.00 mS/m to 25.00 mS/m, and 10.00 mS/m to 16.67 mS/m, respectively. The total width of the two layers is 15 meters approximately. Although, the device can give calibration problems to the user depending on the work frequency, this method is very practical and can be used in many different applications.

I. INTRODUCTION

The electromagnetic induction methods allow the characterization of the subsoil electrical conductivity. It is based on the application of electromagnetic field in which one measures the response of the subsoil through the relation of the primary and secondary fields. There is a wide variety of methods for the electromagnetic exploration, as different sources can be used in order to create the primary electromagnetic field and numerous devices can be used to measure it. One of the most useful methods is the one that considers a dual-loop system: a transmitter and a receiver coil. The modern instruments that belong to this category allow us to directly and quickly measure the apparent conductivity. Apparent conductivity is defined as the conductivity of a homogeneous half-space that produces the same response as measured over the real earth with the same sensor [1]. This method has a wide range of applications: to make salinity maps and evaluate the process, precision agriculture, the characterization of soil water content, geomorphologic and sedimentation purposes, and even planetary explorations in Mars [2]. The different coil configuration allows us to investigate at different depths. They can be horizontal and coplanar (HCP), vertical and coplanar (VCP) or perpendicular (PRP). Other ways to evaluate the behaviour of the subsoil are to change the coil separation (s) and to work at different frequencies. If the separation between the source loop and the receiver loop is somewhat more than five times the diameter of either loop, both loops may be considered mathematically as dipoles [3]. According to the

orientation of the dipole, one can have: vertical magnetic dipoles (VMD) or horizontal coplanar loops (HCP). It must be taken into account that the reference can be either the dipole or the coil orientation, which might cause confusions. In this study, it is applied a low frequency workflow (<200 kHz) [4] as three different frequencies are used: 1, 8 and 16 kHz. It can also be considered a low induction number (LIN). The low induction number B is defined as the coil separation (s) divided by the skin depth (δ), which is defined as the distance in the half-space that a propagating plane wave has travelled when its amplitude has been attenuated to $1/e$ of the amplitude at the surface. The LIN approximation is when $B \ll 1$ [5]. In this paper, I present the theory about the forward problem of multi-frequency modelling and its implementation of dual-loop application in a real situation. As well as, I establish a starting point to solve the inverse problem.

II. THEORY

I. Background

The dual-loop electromagnetic induction exploration method is based on the mutual impedance (Z), which relates the intensity (I) generated by the transmitter loop (Tx) and the voltage (V) applied to the receiver loop (Rx).

$$Z = \frac{V}{I} \quad (1)$$

The transmitter loop carries an electrical current from a specific frequency, which is the re-

sponsible of generating the primary electromagnetic field. It can be assumed that the primary electromagnetic field penetrates vertically into the Earth due to the conductivity difference between the surface and the air ($\sigma_{air} = 0$ and $\sigma_{Earth} > 0$). Then, it induces small eddy currents in the subsoil, which generate the secondary magnetic field. The second loop, located at a distance s from the first one, is in charge of registering the primary and secondary fields. This is why it is called receiver loop. To achieve different penetrations in the subsoil, one can modified the distance, the orientation and the work frequency of the coils [4].

II. Forward modelling

Forward modelling is the simulation of geophysical measurements on a given earth model. In this case, the earth model corresponds to a layered earth over a half-space. It is shown in the next figure.

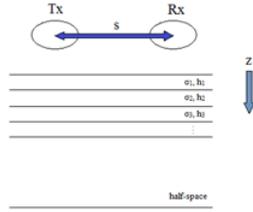


Figure 1: Schematic representation of the geometry of EM sounding for coil orientations in vertical dipole mode (VDM).

As represented in figure 1, σ_1 , σ_2 and σ_3 are the conductivities of the first, second and third layers, respectively, and h_1 , h_2 and h_3 are the corresponding thickness. The resolution of our problem corresponds to solve the Maxwell equation in frequency domain considering the following assumptions:

- The different materials of the layers are not magnetic $\mu \approx \mu_0$.
- Neglecting displacement currents; quasi-static approximation.
- The conductivity of the layers is constant.
- The configuration of each loop is HCP (horizontal and coplanar), which is the same than VDM (vertical dipole mode). They are located at the surface.

Starting with the expressions of intensity of the electrical (\mathbf{E}) and magnetic (\mathbf{H}) field:

$$\begin{aligned} \mathbf{E} &= -\nabla \times \mathbf{F} \\ \mathbf{H} &= -(\sigma + i\epsilon\omega)\mathbf{F} + \frac{1}{\mu\omega}\nabla(\nabla \cdot \mathbf{F}) \end{aligned} \quad (2)$$

Where σ is the conductivity, ϵ is the permittivity and μ is the permeability, ω is the angular frequency and \mathbf{F} is the magnetic vector potential, what is defined as:

$$\mathbf{F} = \frac{i\mu\omega\mathbf{m}}{4\pi r}e^{-\gamma r} \quad (3)$$

Where r is the distance between the point application of the magnetic field and the subsurface point where the magnetic field arrives, and \mathbf{m} is the magnetic moment.

The potential functions and F_i ($i=0,1,2,\dots,N$) for the N different regions in our layered earth must be a solution of the wave equation:

$$\nabla^2 \mathbf{F}_i + \gamma_i^2 \mathbf{F}_i = 0 \quad (4)$$

Where γ_i is the parameter of inductive response of i layer $\gamma_i = \sqrt{i\omega\mu_0\sigma_i}$.

To find the solution of our system, we develop the equation with cylindrical coordinate. We obtain a solution for the wave equation of the following type:

$$\exp \pm [(\lambda^2 + \gamma_i^2)z] J_0(\lambda r) \quad (5)$$

Where J_0 is the Bessel function of zero order and first kind and λ the integration variable.

Applying the respective boundary conditions and, as mentioned, neglecting the displacement currents, it can be achieved the expression of the following magnetic potential vector:

$$F = C \left[\frac{1}{r} + \int_0^\infty R(\lambda, z, \sigma, \omega) e^{-\lambda z} J_n(\lambda r) d\lambda \right] \quad (6)$$

Where C is a constant who depends on the current (I) and the infinitesimal loops area (dA) and J_n the Bessel function of n kind [3].

It is interesting to know the relationship between the primary and secondary fields, which is proportional to the apparent conductivity, as it is the data provided by the loop-loop instruments. The relation between the primary and secondary magnetic fields for vertical dipole mode for the case of homogeneous half-space is given by [6]:

$$\left(\frac{H_s}{H_p} \right)_v = \frac{2}{(\gamma s)^2} \cdot \left\{ 9 - \left\{ 9 + 9 \cdot \gamma s + 4 \cdot (\gamma s)^2 + (\gamma s)^3 \right\} e^{-\gamma s} \right\} \quad (7)$$

Where s is the coil separation.

The low induction number is expressed by:

$$B = \frac{s}{\delta} \quad (8)$$

Where δ is the skin depth defined as $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$

Considering the approximation of low induction number (B), the equation 7 can be rewritten by:

$$\left(\frac{H_s}{H_p}\right)_v = -\frac{18}{(1-i)^2 B^2} (1 - [1 - (1-i)B + \frac{4}{9}(1-i)^2 B^2 - \frac{1}{9}(1-i)^3 B^3] e^{(1-i)B}) \quad (9)$$

And neglecting the terms from the real quadrature and, therefore, focusing on the imaginary part [7], the expression can be simplified at [5]:

$$\left(\frac{H_s}{H_p}\right)_v = \frac{i\sigma\mu_0\omega}{4} s^2 \quad (10)$$

The apparent conductivity provided by the instrument is defined as:

$$\sigma_a = \frac{4}{\mu_0\omega s^2} \text{Im} \left(\frac{H_s}{H_p} \right) \quad (11)$$

Generalising the above expressions by the multilayered earth model, also for the case of horizontal and coplanar loops (HCP),

$$\frac{H_s}{H_p} = 1 - s^3 \int_0^\infty R_0 \cdot J_0(s\lambda) \cdot \lambda^2 \cdot d\lambda \quad (12)$$

Where the term R_0 is the reflection coefficient of the air, which is calculated with the recursive expression starting with $R_{n+1}=0$;

$$R_n = \frac{\frac{\Gamma_n - \Gamma_{n+1}}{\Gamma_n + \Gamma_{n+1}} + R_{n+1} \cdot e^{(-2 \cdot \Gamma_{n+1} \cdot h_{n+1})}}{1 + \frac{\Gamma_n - \Gamma_{n+1}}{\Gamma_n + \Gamma_{n+1}} + R_{n+1} + R_{n+1} \cdot e^{(-2 \cdot \Gamma_{n+1} \cdot h_{n+1})}} \quad (13)$$

The term Γ_n is $\Gamma_n = \sqrt{\lambda^2 + \gamma_n^2}$ and the recursive coefficient n is the number of layers [8].

III. Implementation

For the data evaluation, I generate a MatLab code using the previous mathematical expressions that allows to estimate the Earth model for this study. By using the expression 11, the code allows to calculate the apparent conductivity for a homogeneous medium. Including the expression 10, it gives the imaginary part from the relation between the primary and the secondary fields, for a homogeneous medium with the LIN approximation, and solving the expressions 12 and 13, for the different cases: one, two and three layers. The inputs are the thickness and the conductivities from the different layers. It is necessary to specify the work frequency of the instrument. Bessel functions are included in MatLab and they can be integrated. The resolution of this integral, to obtain the relationship between the primary and

the secondary fields, are not trivial as it needs to integrate an oscillating function with infinite upper limit of integration. To control the integral solved by MatLab, the apparent conductivity calculation was compared using two different expressions: the expression 7 and the general expression 12. On one hand, it was created a simple program to obtain the magnetic field from the expression 7 for a homogeneous medium. And on the other hand, it was calculated the same parameter, but using the general expression 12 for a one layered medium. The values were compared and it is sufficient to assume an upper limit of integration of 50 to solve the integral. The integrand of the functions for 1, 2 and 3 layers was also compared, but imposing the same conductivity values to ensure the behaviours are identical. In order to understand how the subsoil behaves according to the number of layers and their thickness and conductivity, I assessed different synthetic cases for a two-layered medium. I observed that the behaviour of the subsoil is practically independent from the frequency, as expected from the LIN approximation, and when the value of thickness is significant, the conductivity from the first layer is more important. As the thickness values decreases, the second layer behaviour starts to gain importance and the apparent conductivity tends to the numerical value of the second layer.

III. APPLICATION

I. Experimental devices

For data acquisition, I used a device called profiler EMP-400 that contains two coils. It is a lightweight device (4.5 kg) which allows the sweep of large areas in a short period of time. The 'freeway' collection mode allows the user to collect un-gridded data by using GPS coordinates as reference points. It also provides real-time data output via color-coded maps, which enables the user to quickly and easily identify areas of interest on the site. The separation between the coils is fixed ($s=1.21$ m). It is possible to work with three different frequencies simultaneously, from 1 kHz to 16 kHz. The system can be deployed in either the vertical or horizontal dipole mode. Additionally, it is possible to choose to collect the data in a continuous or discrete mode; in this study I used the continuous mode. For each measuring point one can calculate the apparent conductivity [9].

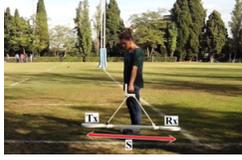


Figure 2: Data acquisition with the profiler EMP-400, with the vertical dipole mode (VCM).

II. Site description

The data acquisition took place on October 31, 2013 in Barcelona. The swept area was the Rugby field of the UB sports centre, which has the following UTM coordinates (WGS 84); N: from 4581400 to 4581500 and, E: from 425.050 to 424.970

The swept area is approximately 1800 m^2 .

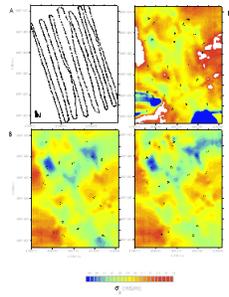


Figure 3: Experimental data. Figure A shows the positions of the measuring points. Figures B, C and D correspond to the apparent conductivity maps for the frequencies 1000 Hz, 8000 Hz and 16000 Hz, respectively. These maps were obtained from the measurements of each frequency for each point shown the figure A and by making an interpolation using the Surfer software.

It can be observed that the graph B has two anomalies; on one hand, there are the white areas that represent negative values of conductivity, but the since conductivity is by definition a positive value, these values should not be taken into consideration because it is due to the calibration of the device [10]. Additionally, one can observe saturated areas (dark blue), where the conductivity exceeds 260 mS/m. It takes place because I unified the conductivity range of values for the three graphs. Therefore, these areas will not be considered for the analysis. Graphs C and D are consistent with each other.

III. Data Analysis

The average values of conductivity obtained from the filed measurement for each data point were plotted through a central profile (NE-SW direction), as it is shown in figure 4.

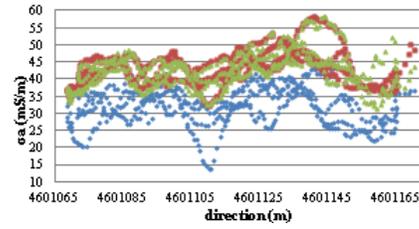


Figure 4: Apparent conductivity projected in a common axis. Values related to 1 kHz, in red, values related to 8 kHz, and in green, values related to 16 kHz.

As one can observe graphically, for the 8 and 16 kHz measurements, the apparent conductivity behaviour is very similar. However, in the 1 kHz case, although one can observe the same type of behaviour, the values of conductivity are lower compared with the other two cases; there is a gap between the conductivity values. As it was mentioned previously on c theory subsections, apparent conductivity is independent of the frequency, so the measurements for these three cases should overlap. To have one numerical reference, one can calculate the average and the statistical mode of all the values. These values are shown in the following table.

Frequencies	1 kHz	8 kHz	16 kHz
σ_a av. (mS/m)	32.283	43.525	43.040
σ_a st. (mS/m)	36.105	45.197	44.415

Table 1: Apparent conductivity; average and statistical mode for three different frequencies.

These numeric values also confirm the gap in the behaviour between 1 kHz frequency and 8 and 16 kHz frequencies. The problem is probably due to a calibration effect of the device used and thus was unable to include those values of 1 kHz successfully [10].

IV. INTERPRETATION

To analyse the characteristics of the medium from the values of conductivity measured, I used the previously mentioned MatLab code. If the apparent conductivity is adjusted for a homogeneous medium case, the half-space conductivity values range is 43.19 mS/m to 45.46 mS/m. For the case of the medium between two layers, the first medium conductivity range is 45.46 mS/m to 50.00 mS/m, and for the second medium is 10.00 mS/m to 10.20 mS/m. The first layer thickness values vary between 15 to 16 meters. For the three layers medium case, the first medium conductivity is over 50.00 mS/m, the second medium conductivity is from 10.00 mS/m to 25.00 mS/m, and the third medium

conductivity is from 10.00 mS/m to 16.67 mS/m. The first layer thickness varies from 5 to 10 meters, and the second layer thickness, from 5 to 20 meters. We know the results obtained with passive seismic methods (Seismic Array and Nakamura techniques) for the same zone analysed in the present study. In one hand, they show the layers' thickness values doing a combined inversion, and on the other hand, they obtained the layer thickness with the Nakamura technique. The values are 15.98 m with combined inversions and, 15.46 m with Nakamura technique. In the two layers case, the first layer thickness value fit well with the values obtained with seismic techniques. In the three layers case, there are more parameters to fit, and their ranges are larger. However, the addition of the two thicknesses is 15 meters in most of the analysed cases, and so, the values obtained with seismic methods are consistent.

V. DISCUSSIONS AND CONCLUSIONS

- This method of dual-loop electromagnetic induction for near-surface, using the EMP-400, allowed the measurements of 3 simultaneously frequencies, which speeds up data acquisition, and allows the user to observe the behaviour in situ, and so adapt the acquisition to its needs.
- This method is very practical and can be used in many different applications. Additionally, it is a very efficient method, as the device does not require any infrastructure.
- Depending on the frequency in which it works, it gets very similar behaviours, almost identical, of the normalized magnetic field. The difference between the frequencies lies in the skin depth. With high work frequencies, it gets less depth but more resolution. And with low work frequency, it gets more depth but less resolution.
- The conductivities of the analysed area are small and so the materials that of the layers have a high electrical resistivity.
- The results are consistent with those found with other geophysical techniques.
- The studied area had no interest in itself, but it is useful to understand the method and how to handle the device. The Mat-Lab modelling code built to interpret this data could be applied to other areas with specific interests.
- The device can give calibration problems to the user; a way to avoid them is by working at frequencies close to the frequency of calibration of the instrument, which is 15 kHz, or by calibrating again the device to the desired frequency.
- For further research, one could apply inversion algorithms to fit automatically the obtained values to the theoretical values of the model, completing the inverse problem. In any case, for the inverse problem resolution, first one must solve the forward problem, so the present work is a first step towards the inversion problem.

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