A thermomechanical cohesive zone formulation describing interface separation in ceramic matrix composites

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Abstract

Ceramic matrix composites (CMCs) are lightweight corrosion-resistant materials that are being engineered for use in aerospace engineering, specifically in jet turbines. In order to counteract the fragility of normal ceramics, ceramic fibers are embedded, in textile-form, into the ceramic. This process changes the failure mechanism of the ceramic and increases its fracture energy; therefore strengthening the material against crack propagation. A crack in the ceramic matrix is deflected at the fiber-matrix interphase and propagates further along the fiber, leading to a separation between the fiber and the matrix. This separation, and the traction that can be transferred from the matrix to the fiber despite the crack, can be modeled with fracture mechanics and the finite element method. Cohesive zones were developed in the mid-20th century to model the behavior of the crack using the finite element method. In this work, a current cohesive zone formulation will be expanded on to include the thermal phenomena present in CMCs at high temperatures. These include conduction, thermo-mechanical coupling, and fiber bridging. These phenomena will then be displayed using numerical examples on a double-cantilever beam.

Introduction

Research and development in aerospace engineering has had a significant focus on developing new materials to increase the lifetime, load-bearing capacity, and economic viability of aerospace vehicles. Materials in the engines of aerospace machines must be able to withstand corrosive environments at high temperatures while being subjected to large mechanical loads. However, the materials must also be light in order to decrease the dead weight of the machine. For this purpose, ceramic matrix composites (CMCs) were developed.

The development of CMCs as a new material has led to the question of how the material should be characterized. Before any material can be trusted to fly safely, many tests must be carried out. To this end, the numerical modeling of CMCs can be used to evaluate and assess their performance without having to conduct expensive tests. The numerical modeling takes place using finite elements, and the failure of CMCs can be modeled using many different methods. For this work, a cohesive zone formulation is used. The current cohesive zone formulation is written to solve delamination problems without mechanical interactions with other environmental influences. However, because the material properties and failure mechanism of CMCs change at high temperatures, it is necessary that this interaction be taken into account for the cohesive zone formulation.

The expansion of the current cohesive zone formulation to include temperature influences in the mechanics can not only be used for CMCs for use in jet engines but can also be useful in other engineering fields that model the thermo-mechanical interaction of materials.

In this work, a cohesive zone formulation for numerical modeling is expanded upon to include thermal phenomena of ceramic matrix composites (CMCs). To begin, an expanded cohesive zone formulation is described, and that formulation is then thermo-mechanically coupled to account for thermal behavior of CMCs. The formulation is then tested to evaluate its mechanical viability and an outlook to possible future research is given.

Methodology

Cohesive zones are elements used in the finite element method to describe the separation of two bodies from one another. Two 2-D bodies, Ω_+ and Ω_-, are connected along a shared boundary by cohesive zone elements. As the bodies are pulled apart, a gap g forms, which consists of the gap in shear direction $\boldsymbol{g_s}$ and the gap in normal direction $\boldsymbol{g_n}$. Similarly, the traction vector at the shared boundary $\boldsymbol{t}$ is broken down into shear traction t_s and normal traction t_n (Fig. 1).

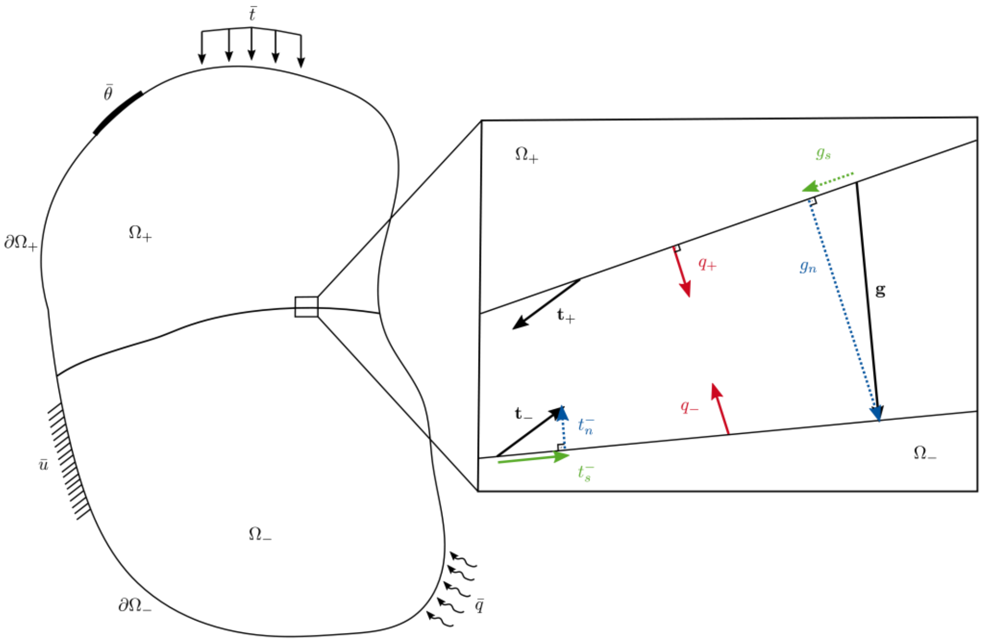

Fig. 1 - Separation of two bodies at a cohesive region with traction, heat flux, and gap vectors.

The free energy per unit area in the cohesive region is defined as

$$\psi_{cz}(g_n, g_s) = \frac{1}{2}(1-d)k_0\lambda^2 + \frac{1}{2}k_p\langle -g_n\rangle^2$$

with
$d =$ scalar damage parameter
$k_0 =$ undamaged stiffness
$\lambda =$ effective separation
$k_p =$ penalty stiffness in case of penetration

The stiffness of the material before any damage takes place, k_0 and the penalty stiffness k_p are constant parameters, whereas the scalar damage parameter d and the effective separation λ are variable depending on the progress of the damage in the cohesive zone. As in [1] this work will assume a quadratic approach for the free energy function. An overview of alternative approaches for potential energy can be found in [2].

The effective separation is defined as $\lambda = \sqrt{\langle g_n \rangle^2 + \beta^2 g_s^2}$ and with the variable β determines the contribution of the shear gap to the effective gap in the cohesive zone. Macaulay brackets ensure that negative gaps in normal direction (penetration of the body) are prevented. Additionally, if $g_n < 0$, the penalty stiffness k_p is implemented to counteract the penetration of the bodies. The effective traction $\boldsymbol{t}$ is defined as

$$\mathrm{t} = \frac{\partial \psi_{cz}}{\partial \lambda} = \sqrt{t_n^2 + \beta^{-2} t_s^2} = (1-d) k_0 \lambda$$

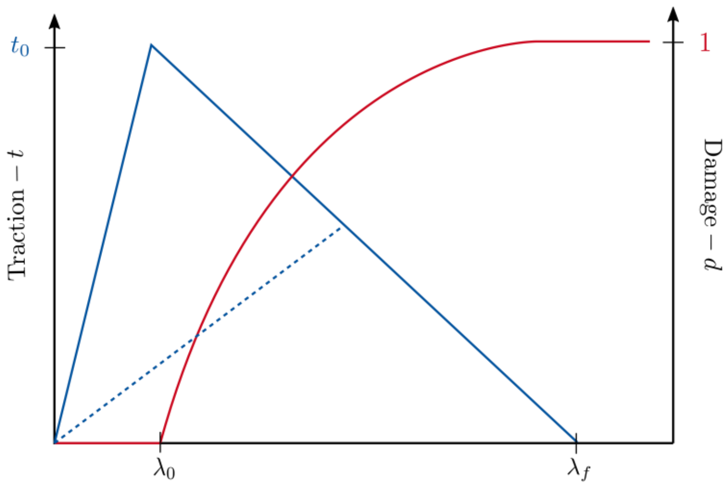

Fig. 2 - Bi-linear traction-separation law.

A TSL is used in order to determine the separation, and damage, in a specific element based on a given traction. For this work, a bi-linear TSL will be considered (Fig. 2) due to its advantages in modeling the damage in CMCs [3]. The separation at maximum strength t_0 is identified as λ_0. The damage parameter is defined as

$$d = \begin{cases} 0 & \text{if } \lambda < \lambda_0 \\ \dfrac{\lambda_f}{\lambda_f - \lambda_0} \dfrac{\lambda - \lambda_0}{\lambda} & \text{if } \lambda_0 < \lambda < \lambda_f \\ 1 & \text{if } \lambda_f < \lambda \end{cases}$$

The contribution of the cohesive zone to the weak form of equilibrium can be found in [4]. Similarly, the discretization of the cohesive zone formulation for the finite element method is defined in [1].

In order to expand the equilibrium to include conduction, a heat flux in the cohesive zone is introduced as

$$q_c = -(k_c + k_a)g_\theta$$

The material thermal conduction coefficient k_c and the thermal conduction coefficient of air k_a are multiplied by the thermal gap for the heat flux, which is assumed to be solely normal to the interface. The thermal conduction coefficient is considered because of the porous structure of the material and the air pores which develop in the gap [4]. Additionally, g_θ is defined as the temperature jump across the gap from one side of the interface to the other. Ideally, the thermal conduction coefficient of the interface is the same as the coefficient for the fiber and matrix. This will result in perfect conduction in an undamaged state.

To account for thermo-mechanical coupling, the mechanical part of the free energy equation is expanded, and a new traction is derived as follows

$$\psi_{cz,mech}(g_n, g_s) = \frac{1}{2}(1-d)k_0(1-c_c\theta_m)\lambda^2 + \frac{1}{2}k_p\langle -g_n\rangle^2$$

$$\mathbf{t} = \frac{\partial \psi_{cz}}{\partial \lambda} = (1-d)k_0(1-c_c\theta_m)\lambda$$

Similarly, the heat flux will be expanded to account for mechanical damage of the cohesive zone which will affect the heat transmitted from one interface to the other. The expanded heat flux is defined as

$$q_c = -[(1-d_\theta)k_c + k_a]g_\theta$$

where a damage term similar to that in the mechanical part is introduced to the thermal part of the free energy equation and is defined as

$$d_\theta = \begin{cases} \dfrac{\lambda}{\lambda_f} & \text{if } \lambda < \lambda_f \\ 1 & \text{if } \lambda \geq \lambda_f \end{cases}$$

Results

In order to demonstrate the effects of the new formulation, a double-cantilever beam as shown in Fig. 3 is used. Conduction is shown by a simulation with a constant temperature of $0°\,C$ along the lower edge of the beam and a constant temperature on the upper edge of the beam seen in Fig. 4. To display the thermo-mechanical coupling of the mechanical part, the beam is pulled apart with a given displacement at varying temperatures and the resulting force is measured. For this, a constant temperature along the entire beam is brought on and then the beam is pulled apart at the bottom-right and upper-right corners linearly over time. The separation will stay constant; however, the temperature of the beam will be varied for each test as seen in Fig. 5. Material parameters are given in Tab. 1.

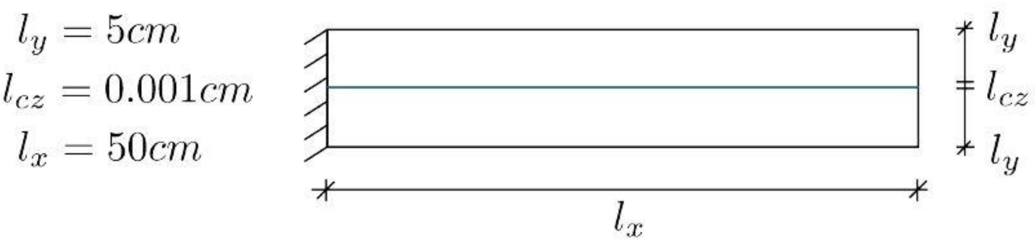

Fig. 3 – Double cantilever beam with measurements

E	ν	α_T	k	c
GPa	$-$	$1/K$	W/mK	$J/kg \cdot K$
0.38	0.2	0	1000	0

t_0	λ_0	λ_f	β	k_c	k_a	c_c
GPa	cm	cm	$-$	W/mK	W/mK	$1/K$
0.04	0.1	1	1	10000	200	0.001

Tab. 1 – Material parameters of the beam and the cohesive zone elements

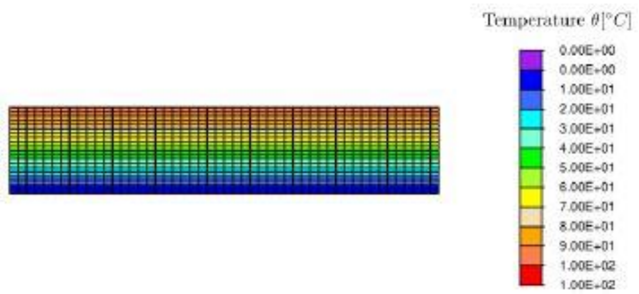

Fig. 4 – Conduction at $\Delta\theta = 100K$

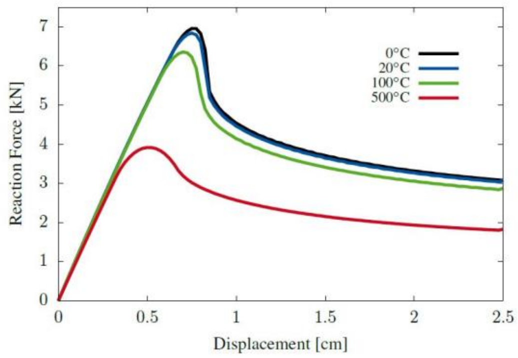

Fig. 5 – Force-Displacement curve.

Conclusion

The aim of this work is to expand an existing cohesive zone formulation as proposed by [1], taking the material behavior of SiC/SiC CMCs at varying temperatures into account, to include conduction and thermo-mechanical coupling. Conduction is included in the formulation by expanding the traction vector with a heat flux for the cohesive zone and expanding the gap vector to include a thermal gap, or the temperature gradient from one side of the cohesive zone to the other. The heat flux is defined with respect to the new thermal gap and the interfacial thermal conduction coefficient. The thermo-mechanical coupling is needed to couple not only the mechanical part with the temperature, but also the thermal part with the mechanics. To achieve this, a material stiffness softening term is added to the mechanical part. This softening term reduces the material stiffness based on the temperature at the mid-plane of the cohesive zone and the experimentally defined variable cc of the interface. The heat flux is then expanded with a damage parameter, reducing the thermal conduction coefficient of the interface with increasing damage.

Possible further research could include the accurate characterization of material parameters for the interface through physical tests, temperature dependence of other material parameters, and the addition of fiber bridging, another important thermal phenomenon of CMCs.

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